COMPUTER-AIDED DESIGN OF AN AUTOMOTIVE EPICYCLIC GEARBOX

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CERTIFICATE

This is to certify that the thesis entitled "Computer Aided Design of an Automotive Gear Box" is a record of work carried out under my guidance and that it has not been submitted elsewhere for a degree.

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ABSTRACT

The problem of designing an automobile transmission - an epicyclic gear box - is formulated and solved as an analytical problem and also as a constrained minimization mathematical problem. The design constraints that are imposed on the problem ensure that the number of teeth on any pinion are without undercut (or interference), the induced stress due to dynamic load on the teeth is less than the allowable stress, when transmitting the maximum torque, the ratio of face width and circular pitch is within limits, the number of planets used ensuring assembly of gears in each train; all the gears in each train remaining coaxial. For the allowable maximum torque, the overall size i.e. the volume is minimized, allowing a little variation in the desired speed reduction.

To show the usefulness of the proposed procedure, a comparison between the existing design of a transmission and that of the optimised one is made.

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LIST OF SYMBOLS

Suffixes 1, 2, 3 and 4 refer to the First, Second, Third and Reverse Trains respectively.

S_i, SN_i, NS_i = No. of teeth of sun gears; i = 1,2,3,4

A_i, AN_i, NA_i = No. of teeth of Annuli, i = 1,2,3,4

P_i, PN_i, NP_i = No. of teeth of planet gears, i = 1,2,3,4

SK_i = Ratio of face width to the circular pitch or Face width factor

T = Maximum torque in kg-m

FEND = Endurance stress of the material of the gears

$V, VELS_i$ = Pitch line velocity of sun gears in m/sec (or ft/min)

b_i = Face width of the gears, i = 1,2,3,4

$(XR)_i$ = Speed reductions, i = 1,2,3,4

f = Static stress in kg/mm^2 of gear materials

f_{st} = Static stress in lb/sq.in of gear materials

W_i = Total applied load per tooth in lbs, i = 1,2,3,4

Wd_i = Total dynamic load in lbs, i = 1,2,3,4

Wb_i = Bending load in lbs, due to endurance stress

c_i = Deformation factor, i = 1,2,3,4

e_i = Error in action in inches, i = 1,2,3,4

p_i = Circular pitch of gear teeth, i = 1,2,3,4

y_i = Tooth form factor, i = 1,2,3,4

$W(\bar{x})$ = Objective function

$P(\bar{x})$ = Penalty function

$[H]$ = Hessian Matrix or any positive definite matrix

g_i = Constraints, $i = 1,2,\ldots,25$

$\equiv$ G(I)

GW(I) = Gradient of objective function

GP(I) = Gradient of Penalty function

R = Parameter weighing the penalty term of P-function

$\bar{s}$ = Direction vector at the i^{th} iteration

β = Step length

λ_i = Langrange

$(DIAS)_i$ = Diameter of sun gears, $i = 1,2,3,4$

$(DIAA)_i$ = Diameter of annulus gears, $i = 1,2,3,4$

$(AM)_i$ = Module gears in the train, $i = 1,2,3,4$

$(PT)_i$ = No. of planets in the gear train, $i = 1,2,3,4$

CHAPTER I

INTRODUCTION

The characteristics of a piston engine which is universally used in the passenger cars are such that it is necessary to make available a choice of reduction ratios, so that sufficient engine power may be obtained over a fairly wide range of road speed. The lowest ratio must be such that, starting from rest may be accomplished on the steepest gradients which are likely to be encountered; the higher ratio must provide for suitable acceleration to be maintained while speed increases and an appropriate top speed. These functions can be provided for by a manually operated gear box utilizing a conventional type layshaft gears or epicyclic gear trains.

1.1 EPICYCLIC TRAIN

An epicyclic train Fig. 1.1 in its simplest form consists of three elements; a sun gear, an annulus (or ring gear) and a carrier, the later supporting two or more planet wheels which mesh with both the sun gear and the annulus.

A difference in velocity ratio and/or direction of rotation may be achieved by holding stationary one of these elements and turning another, when the third member will respond relatively. The various speed ratios which can be obtained are described in Appendix B.

Epicyclic gears (planetaries) were widely used in the early days of motor cars and were introduced by F.W. Lanchester in about 1899. These were superseded, with the notable exception of the Model T-Ford, by sliding layshaft gears (credited to Panhard in 1904). Again this type of gear was used in the well known Wilson pre-selective gear box. In 1940, again a preference for the epicyclic trains is shown for the automatic transmissions.

Epicyclic gears are particularly useful in automatic transmissions for the simple reason that changes from one ratio to another can be made without closing the throttle or losing traction, as there is no axial engagement or disengagement of gear teeth or dogs.

Other advantages of epicyclic gears are as follows :

1. An epicyclic gear train is extremely compact when compared with a layshaft train capable of transmitting the same torque- in fact, the length of an epicyclic gear train need be slightly more than the width of one gear face, the planet pinion bearings being contained within the pinion width.

2. The tooth load in an epicyclic train is shared since there are at least two (usually three) equally spaced planets, whereas, in a layshaft train, the power is transmitted by a single pinion.

3. A further advantage of having several planets is the balance, thus achieved, of the loadings on the bearings of the

three elements, whereas the shaft bearings of a layshaft train are heavily loaded.

Thus an epicyclic train is inherently quieter since tooth loads are shared and, more important, the pitch line velocity is reduced owing to the rotation of the carrier arm.

4. The torque difference between input and output members of any gear train is inversely proportional to the speed difference. In a layshaft gear train, if the output speed is, say, half the input speed, the output torque will be twice the input torque. The difference in torques is that reacted by the casing, which must be restrained from rotation. Now, to see the basic relationship between an epicyclic train and layshaft gearing, the reaction of the casing of the layshaft gear box will be the same as that on the carrier of the epicyclic train.

5. A layshaft gear arrangement can, with a given set of parts, provide only two ratios, but a given epicyclic train can provide six ratios*: two reducing ratios, two increasing ratios and two reversing ratios.

There are usually two limiting factors in the simple gear train design; one is the minimum diameter of the planets and the other is the minimum diameter of the sun wheel, which is usually imposed by the output or input shaft running through the centre of it. It can be easily shown that it is impossible

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* Details given in Appendix B

to obtain a ratio of 2:1 with a simple train.

The compound trains are ideal for overcoming the simple planet ratio limitations and offers a great deal of freedom to the designer. By varying the ways in which two gear sets are compounded and selecting input and output members, it is possible for the designer to obtain gear trains of equal ratio, but with high tooth load in one train and low in the other.

For a given output speed, if the sun wheel is the output member, its speed will be low and so will transmit a high torque. Therefore the sun wheel should be avoided as the output member wherever possible as it results in higher tooth loads and a consequent increase in the size of the whole train.

6. The gear ratio changes are made by applying hand brakes to cylindrical drums integral with the annulus. These occupy far less space as compared to sleeves, levers, etc. used in a layshaft gear box.

7. Another important advantage which epicyclic gears have over layshaft gears when considered for automatic transmissions - the ability to change from one ratio to another without loss of torque transmission - is achieved by suitable clutches or brakes associated with the reaction members, one of which is released as another is engaged.

8. Since a torque balance must exist when transmitting power, it will be apparent that the torque in the brake or clutch

holding a reaction member is the difference between input and output torque. Thus for large gear steps, the reaction torque is relatively high and it is a common practice to use a one-way clutch (or free wheel) or a self energizing brake band. For lower ratios, where the reaction torque is small, plate or cone friction clutches are more commonly employed. The potential speed of a reaction member is inversely proportional to the torque required to hold it stationary i.e. a reaction member which, by virtue of the gearing, would run at a high speed with the output member stalled, requires only a small torque to hold it stationary. The actual energy which is absorbed (and dissipated as heat) in a friction clutch or brake during a shift is related to the torque transmitted and the ratio step and is independent of the potential speed difference.

1.2 LITERATURE SURVEY

Quite a large number of papers on epicyclic gearing have been published. E.F. Obert[1] describes the method of calculating input and output speed ratio from torque relationships. L.A. Graham[2] gives the principles underlying variable speed differential drives. A Verhoeef[3] enlists advantages of some transmission mechanisms to be used in regulating transformers operated by remote control and for driving stirring devices in chemical experiments, when transmission ratio is high and power to be transmitted is low.

C.Carmichael[4] relates the unlimited variety of possibilities in providing speed ratios between driving and driven members and simplification of calculation of speed ratios by routine procedure. V.Francis[5] describes the possibility of using more than three planet pinions and formulas for spacing of pinions in the carrier. R.H.Macmillan[6] gives the extra ordinary diversity of trains possible even when only five or six wheels are employed. A.Balogh[8] gives the simplified formulas for calculation of velocity ratio of epicyclic trains. M.A. Plint[9] describes the importance of correct choice of numbers of teeth in sun wheel and annulus and equations for designing simple epicyclic trains with three pinions. W.H. Maun[10] gives a method for working out simple epicyclic gear trains. S.Rapport[11] explains the method of equal arcs for finding ratio of input to output in a planetary gear train. D.B. Welbourn[12] gives a simplified method of calculation due to Beyer and Kutzback. R.N. Abild[13] describes a simplified method for designing and selection of epicyclic gear systems, where choice of system is possible. E.I. Radzimovsky[14] expresses the method of finding efficiency, speed and power in following four advanced systems of planet gear, having complex kinematic relationships, two inputs shafts or two output shafts. R.J. Willis[15] explains a method which permits rapid feasibility studies of epicyclic gears. W.A. Tuplin[16] enlists the practical possibility and

limitations of four-gear epicyclic trains in drive applications and method of selecting the number of teeth of different gears so that efficiency is maximum at high velocity ratios. R.M. Couklin and James Ballmer[17] compute the forces and speeds for simple and compound gear trains, based on summation energy equations. H.G. Laughlin and A.R.Holowenko[18] have developed the general design equations as functions of input power and expressions for functions are also established in terms of defined velocity ratio across entire system and across control circuit. H.C. Town[20] relates the design of planetary gear and method of tabulating movements and analysis laws governing speed and direction of rotation.

Most of these papers deal with the analysis of gearing of simple epicyclic trains. In no case any attempt is made to minimise the space occupied by coupled epicyclic trains which is characterised by two or more single trains, the rotating members of which are coupled together. Such a train is an indispensable sub-assembly in an automobile. It is certainly an advantage if the gear box designed occupies the least possible space, at the same time satisfying all the necessary and essential requirements.

CHAPTER II

DESIGN PROBLEM

The system to be designed is a transmission gear box for an automobile. The function of this system is to deliver four forward speeds and one reverse speed to the output shaft (propeller shaft) from a single speed diesel engine for which the maximum torque is specified at a specific speed. The total volume of the gear box, excluding operational clearances between adjacent gears is to be as small as possible.

To show the effectiveness of the synthesis procedure, a live problem is chosen. In one of the U.K. make buses, an epicyclic gear box is used. The gear box has the following specifications :

Max. torque 62.1 kg-m at	1100 rpm.
First speed reduction	1 : 4.28
Second speed reduction	1 : 2.43
Third speed reduction	1 : 1.59
Top speed	1 : 1
Reverse speed reduction	1 : 5.97

Our problem is to synthesise an epicyclic gear box of the above specifications in such a manner that of all the existing possible designs, the chosen one will occupy the least space.

2.1 DESIGN CONCEPT

There can be many schemes to satisfy the above specifications, but the scheme selected is that which is

actually being used in the example problem so that the synthesis made finds a good comparison. This scheme is shown in Fig. 2.1. The input is the engine side and the output is the propeller shaft. There are four epicyclic trains of gears so that three velocity reduction ratios, one direct speed (these four being forward speeds) and one reverse reduction ratio, are obtained. The top gear drive is provided by directly connecting the input shaft to the output shaft by using a multiplate clutch. The first speed planet gear carrier acts as the driving member to the output shaft for all the forward speeds and the planet gear carrier of the reverse gear train acts as the driving member for the output shaft for the reverse gear.

2.2 OPERATION OF GEAR BOX

FIRST GEAR - The first gear speed is obtained by applying a brake to the first gear annulus so that it is held stationary. The engine is turning the main sungear so that the planet gears will be rolling round with it. This carrier is fixed to the output shaft through splines. Therefore, it imparts motion to the output shaft and so to the rear axle.

SECOND GEAR - The second gear speed is obtained by holding the second train annulus stationary by applying the brake to it. The main sun gear being turned by the engine, causes the planet gears to revolve and turn their carrier. This carrier is connected to the first gear annulus which therefore turns, speeding up the rotation of its planet gears

and carrier. In this case the output shaft runs faster than the first gear i.e. less reduction is achieved.

THIRD GEAR - The third gear speed is obtained by holding the third gear brake drum, which is integral with the sun gear running free on the engine shaft. The annulus of the third gear train is integral with the second gear planet carrier. The third gear train planet carrier is connected to the second gear annulus and drives it in the same direction as that of the engine shaft i.e. increasing its speed. So the drive is taken back through the second gear planets and carrier and the first gear annulus, both of these are speeded up. This results in speeding up of the first gear planets and carrier which imparts faster motion to the propeller shaft. In other words by interconnecting the second and third planetary trains, an increase of speed is obtained at the first gear annulus, which increases the speed of planets and carrier.

TOP GEAR - In top gear, all the trains are locked together so that they revolve as one solid cylinder driving the output shaft at the engine speed. This is brought about by releasing all the brakes and actuating the clutch which locks the sun gear of the third train to the driving shaft, through the drum. This causes all the sun gears to revolve at the same speed i.e. the engine shaft speed.

REVERSE GEAR - The first gear annulus is connected to the sun gear for the reverse gear train and hence drives it in a direction opposite to that of the engine. When the

brake is applied to the reverse gear annulus, the planets of this train carry with them the planet carrier in the opposite direction to the engine shaft rotation. As the planet gear carrier is directly connected to the output shaft, the direction of rotation of the output shaft is reversed.

2.3 DESIGN PARAMETERS

It is convenient to consider three distinct types of parameters - independent variable, constant and dependent variable. The independent variables are those that the designer is free to select independently in his design synthesis. Constants are those quantities which are imposed by design requirements or have been decided upon in the first phase of the design process. Together, the independent variables and constants completely describe a design.

Dependent variables are quantities introduced to simplify the mathematical relationships involved. Though the mathematical model can be constructed without the use of these dependent variables, but they are needed to simplify the mathematical relationships.

2.4 DESIGN SPACE

If the number of independent design variables is n for a particular design problem, then every possible design can be thought of as being represented by a point in an n-dimensional space. This space is called design space.

2.5 CONSTRAINTS

The constraints are limits imposed on individual parameters or on group of parameters, in order to insure their physical realizability or their compatibility with the rest of the system and the environment.

2.6 SYNTHESIS

The synthesis of a design is the process which leads to the selection of a point in the design space. Any point in the space is called a 'design' even if this gives impossible configurations. The analysis is carried out to know whether the design is worthwhile or not.

2.7 OBJECTIVE FUNCTION

If the designer desires to find the best design among all those in the feasible region, he must define a function $f(\overline{x})$, called the objective function whose value is a measure of the merit of the design. Conventionally f is designed such that the better the design, the lower its value. The best feasible design is the one for which $f(\overline{x})$ is minimum for

$$g_i(\overline{x}) \geq 0 \qquad \text{for } i = 1,2,\ldots\ldots,m$$

This formulation of the problem is called a mathematical programming problem.

CHAPTER III

COMPUTER AIDED DESIGN
(DESIGN ANALYSIS AND SYNTHESIS)

In this chapter, the important design constraints are discussed. For the sake of convenience in mathematical relations, all these constraints are expressed as inequality constraints. These are obviously functions of design parameters.

The subscripts 1, 2, 3 and 4 refer to first, second, third and reverse gear trains respectively.

3.1 DESCRIPTION OF THE PROBLEM

For a given amount of energy to be transmitted it is quite obvious that the best system is that which is smallest in size. Decrease in dimensions leads to the decrease of weight, economy of material, reduction of cost, less vibrations, friction and wear.

The volume is calculated by considering all the four gear trains placed side by side, as the space for operational clearances, carrier plates (arms), etc. is fixed and cannot be reduced. The volume therefore depends on the outside diameters and face width of the annuli of the gear trains.

3.2 CONSTRAINTS

The calculations are based on the following considerations or constraints :

3.2.1 Condition of Coaxiality

According to this condition, the axes of the central gears must coincide. This facilitates tapping energy from this mechanism or imparting energy to this. In Fig. 3.1, the arm A is the inner and central gear 4 is the given element. Obviously in order that the gears 4 and 1 are coaxial

$$r_1 + r_2 = r_4 - r_3 \tag{3.1}$$

where r_1, r_2, r_3 and r_4 are the radii of pitch circles of gears 1, 2, 3 and 4 respectively.

3.2.2 Condition of Neighbourhood

To minimize the load per planet and proper balancing of the arm, generally not one but a few planets are fixed to the same arm. The planets are generally placed at equal angular distances apart and they are placed on the same plane perpendicular to the axis of the gear 1 as shown in Fig. 3.2. The load per planet gear is minimum or the number of planet gears is maximum when their addendum circles just touch each other as shown in gear 2 and 2'. From the triangle ABC, so that the addendum circles do not touch, the condition is

$$2(r_1 + r_2) \sin \frac{\pi}{Q} > 2 \cdot ro_2$$

where ro_2 is the radius of the addendum circle of the planet gears and Q is the number of planet gears on the arm

$$\text{or } (z_1 + z_2) \sin \frac{\pi}{Q} \geqslant z_2 + 2$$

$ro_2 = r_2 + m$, where m is the module

$$\sin \frac{\pi}{Q} \quad \frac{z_2 + 2}{z_1 + z_2} \tag{3.2}$$

3.2.3 Condition of Assembly

If Q is the number of equally spaced planets, the numbers of teeth in the concentric gears must satisfy the condition of assembly

$$\frac{z_1 + z_3}{Q} = \text{Integer} \tag{3.3}$$

where z_1 and z_3 are the numbers of teeth in the sun gear and annulus gear respectively.

The proof of this is as in Fig. 3.3. In this figure let P be the first planet to be assembled on the carrier R, the arrows indicate two teeth on P and A in engagement. If it is to be possible to assemble a second planet spaced at $\frac{360}{Q}$ degrees from the first, it follows that if R is rotated through $\frac{1}{Q}$ revolution with S fixed, the planet P must cause the gear A to rotate by an amount which brings another tooth into exactly the same position as that which originally engaged with P at its starting point.

In $\frac{1}{Q}$ revolution of carrier R, the concentric gear A will move through

$$\frac{1}{Q} \cdot \frac{\left(-\frac{z_3}{z_1} - 1\right)}{-\frac{z_3}{z_1}}$$

$$= \frac{z_3 + z_1}{Qz_3} \text{ rev.}$$

For assembly of the next planet to be possible, this must be a multiple of one pitch or $\frac{1}{z_3}$ revolution of A i.e.

$$\frac{z_3 + z_1}{Qz_3} = k \times \frac{1}{z_3}$$

or $$\frac{z_3 + z_1}{Q} = k = \text{Integer.}$$

3.2.4 Stress Constraint

The induced stress in the material of the gears at the critical section of a tooth, should always be less than the allowable stress as explained below :

3.2.4.1 Tangential Tooth Load

For a given horse power to be transmitted by a pair of gears, the tangential tooth load W is given by the following horse power equation :

$$\text{H.P.} = \frac{2\pi NT}{4,500} \tag{3.4}$$

where N = R.P.M. of the gear

T = Torque in kg-m

= WR

W = Tangential tooth load in kg.

R = Radius of the pitch circle of the gear in m

3.2.4.2 Beam Strength of Teeth

The force which the tooth of one gear transmits to the other causes bending stresses in the teeth. The following Lewis equation gives the tangential load which the tooth will carry in beam action :

$$W = S_1 bYP \qquad (3.5)$$

where S_1 = Maximum bending stress in kg/cm^2

b = Width of the tooth in the axial direction in cm

Y = Lewis form factor which depends on the number of teeth in the gear and the system of gearing used

$= 0.154 - \frac{0.912}{z}$ for 20° pressure angle.

This form factor can also be found from tables on curves for a given number of teeth z.

p = circular pitch in cm.

3.2.4.3 Dynamic Gear-Tooth Load

The load applied to the gear tooth is greater than the transmitted load based on horse power. In general, the faster the gears are running, the more shock due to tooth errors and the more dynamic effects due to unbalance and torque variations in the driving or driven parts.

A considerable amount of research has been carried out to determine the amount of dynamic gear tooth loads. A research committee of the American Society of Mechanical Engineers - under the Chairmanship of E.Buckingham[30] published the first authoritative work on dynamic loads. A simplified

formula giving a quick but approximate calculation of dynamic load, Wd is :

$$Wd = W + \frac{0.05V(bc + W)}{0.05 + \sqrt{bc + W}} \tag{3.7}$$

where V = pitch line velocity in ft/min

b = face width in inches

c = deformation factor - from Tables (30) or as given below :

3.2.4.4 Determination of Deformation Factor c

It is necessary to have some measure of the error in action of the gears. The noise of operation is usually a very good test of the accuracy of gears. The following table[29] gives some measure of the order of accuracy required at different pitch line velocities and should serve as a guide for the selection of the proper class of gear to meet specified speed conditions. The values shown in the table should keep the noise of operation and the intensity of dynamic load within reasonable limits.

V ft/min	Error	V ft/min	Error
250	0.0037	2,250	0.0013
500	0.0032	2,500	0.0012
750	0.0028	2,750	0.001
1,000	0.0024	3,000	0.0007
1,250	0.0021	3,500	0.0007
1,500	0.0019	4,000	0.0006
1,750	0.0017	4,500	0.0006
2,000	0.0015	5,000 and over	0.0005

Corresponding to these errors, the value of c is known from the table given below :

VALUES OF c

Gear Material	Tooth Form	Error in action in inches					
		0.005	0.001	0.002	0.003	0.004	0.005
Steel on Steel	20°Full depth	830	1,660	3,320	4,980	6,640	8,300

3.2.4.5 Margin of Safety

The static beam strength of the tooth should always be greater than the dynamic load as :

For steady loads, $W_b = 1.25$ Wd

For pulsating loads, $W_b = 1.35$ Wd

For shock loads, $W_b = 1.50$ Wd

Many gear designers[31] have found that gears do not have to be designed to carry as heavy a dynamic load as given by Bückingham equation which gives the dynamic loads in the order of 135 to 175 percent.

Automative gears are usually fully hardened. Most of the time, they run at quite low torque loads. Frequently their operation at full torque amounts to less than 100,000 cycles. Tooth strength is usually more important consideration than wear. Dynamic overload due to tooth errors is usually not too important because the parts are light, the shafts are limber and the speeds are low. Misalignment effects are not too serious because the face widths are narrow. Frequently the ends of the teeth are relieved on the tooth is crowned. This prevents serious trouble due to possible concentration of the load at the ends of the teeth. So the above dynamic load type of calculations may be used as an alternative to the use of the velocity factor described below :

3.2.4.6 Dynamic Factor - Barth's Equation

The velocity factor is intended to account for shock loading which results when accurate teeth are run together at high speed. For spur gears the velocity factor or dynamic factor C_v is given by :

$$C_v = \frac{3.01}{3.01+V} \qquad (3.8)$$

where V = periferal velocity at pitch circle in m/sec.

The allowable stress is then modified as given below in the Lewis equation :

$$S_a = C_v S_1 \tag{3.9}$$

The Lewis equation is therefore written as

$$W = S_a Y b p \tag{3.10}$$

3.2.5 Velocity Ratio Constraint

The final velocity ratio can have a maximum variation of 2 percent from the nominal values specified. This limit is based on experience.

3.2.6 Interference Constraint

To avoid interference or undercut in the teeth of the pinions, the minimum number of teeth should not be less than 14 but according to Buckingham[29] for epicyclic gears the minimum number of teeth should preferably be 16, i.e.

$$z_i \geqslant 16 \tag{3.11}$$

3.2.7 Planet Teeth

The number of teeth, P, on the planets should be equal to $\frac{A-S}{2}$ and $(A-S)$ should be an even number[28].

3.2.8 Sun and Planet Centre Distance

The sun and planets should be meshed at a centre distance corresponding to $(S + P + 2)$ teeth[28] to avoid interference.

3.2.9 Annulii Interference

The teeth of the annulii should be given a correction coefficient of - 0.5 to avoid fillet interference when assembling[28].

3.2.10 Tip Interference

To avoid tip interference between the internal gear (annulus) and pinions (planets), the size of the pinions must be sufficiently smaller than the size of the internal gear[29]

3.2.11 Face Width Restriction

According to ISI - 3681-1966 the ratio of face width and circular pitch, k should be between 2 and 5. The best values considered from experience is 3 to 4.

3.2.12 Standard Modules

The module is defined as the ratio of the pitch diameter to the number of teeth. This must be always positive. Though the modules are standardised, these will be considered as continuous variables. It is usual, from experience to have the values of modules not greater than 10. The preferred values are : 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6,6.5,....

(3.12)

3.2.13 Size of Annulii

The diameter of the annulus of first, second, reverse and the drum for the third gear and top speed should not have much variation to get a compact gear box. So the variation on the diameter allowdd to the annulii of second and reverse

gear train is restricted to 5 percent with respect to the annulus of first gear train. As the third gear train annulus is enveloped by the second gear annulus, allowing free movement of each, so the variation allowed is such that the diameter of third gear annulus is 15 to 5 percent lower than the diameter of the first or second gear annulus i.e.

$$1.05\ DIAA1 \geq DIAA2 \geq 0.95\ DIAA1 \quad (3.13)$$

$$0.95\ DIAA1 \geq DIAA3 \geq 0.85\ DIAA1 \quad (3.14)$$

$$1.05\ DIAA1 \geq DIAA4 \geq 0.95\ DIAA1 \quad (3.15)$$

3.2.14 Machining Cost Reduction

To reduce the cost of manufacture, the sun gears of the first and second gear trains are made common in one integral unit which implies that the module will be the same for the first and second gear trains.

3.3 ANALYSIS FOR NUMBER OF TEETH

By considering the condition of coaxiality and the module of the gears in a train to be the same, for the given velocity ratio $(XR)_1$, the tabulation method is used for calculating the relations between the teeth of various gears.

3.3.1 Direct Speeds

3.3.1.1 First Train (Fig.3.4)

With annulus A_1 held stationary with brake

	R_1	A_1	S_1
Step 1	+1	+1	+1
Step 2	0	-1	$+\frac{A_1}{S_1}$
Result	+1	0	$1+\frac{A_1}{S_1}$

$$\therefore (XR)_1 = 1 + \frac{A_1}{S_1}$$

$$\text{or} \quad A_1 = \left[(XR)_1 - 1\right] S_1 \tag{3.16}$$

$$P_1 = \frac{A_1 - S_1}{2}$$

$$= \left[\frac{(XR)_1 - 2}{2}\right] S_1 \tag{3.17}$$

3.3.1.2 Second Train (Fig.3.5)

Annulus A_2 is held stationary.

For train 2

	R_2	A_2	S_2
Step 1	+1	+1	+1
Step 2	0	-1	$+ \frac{A_2}{S_2}$
Result	+1	0	$1 + \frac{A_2}{S_2}$

Now $A_1 = 1 = R_2$

$S_1 = S_2$

R_1 revolves at x revolution

	R_1	$A_1(=R_2)$	$S_1(=S_2)$
Step 1	x	x	x
Step 2	0	1-x	$-(1-x)\frac{A_1}{S_1}$
Result	x	+1	$x(\frac{A_1+S_1}{S_1})-\frac{A_1}{S_1}$

Equating $S_1 = S_2$

$$\frac{A_2 + S_2}{S_2} = x\left(\frac{A_1+S_1}{S_1}\right) - \frac{A_1}{S_1}$$

$$\text{Or} \quad x = \left[\frac{A_2+S_2}{A_1+S_1}\right]\frac{S_1}{S_2} + \frac{A_1}{A_1+S_1} \qquad (3.18)$$

$$\therefore \text{ Gear Ratio } (XR)_2 = \frac{A_2+S_2}{S_2 . x}$$

$$x = C22$$

$$A_2 = S_2 \times (XR)_2 - S_1$$

$$= S_2(C22)\ (XR)_2 - S_1 \qquad (3.19)$$

<u>3.3.1.3 Third Gear Train</u> (Fig.3.6)

S_3 is held stationary

	R_3	S_3	A_3
Step 1	+1	+1	+1
Step 2	0	-1	$\frac{S_3}{A_3}$
Result	+1	0	$1+\frac{S_3}{A_3} = G_3$

For train 2 $A_2 = R_3$, $R_2 = A_2$

	R_2 (= A_3)	A_2 (= R_3)	S_2
Step 1	G_3	G_3	G_3
Step 2	0	$1 - G_3$	$-(1-G_3)\frac{A_2}{S_2}$
Result	G_3	+1	$G_3(\frac{A_2-S_2}{S_2})\frac{A_2}{S_2}$

In train 1 speed be x-unknown

	R_1	A_1 (= R_2)	S_1 (= S_2)
Step 1	x	x	x
Step 2	0	$G_3 - x$	$-(G_3 - x)\frac{A_1}{S_1}$
Result	x	G_3	$x(\frac{A_1+S_1}{S_1}) - \frac{G_3 A_1}{S_1}$

$$= 1 + \frac{S_3}{A_3} \qquad (3.20)$$

By equati : the results of S_1 and S_2

$$G_3 \left(\frac{A_2 + S_2}{S_2}\right) - \frac{A_2}{S_2} = x \left(\frac{A_1 + S_1}{S_1}\right) - \frac{G_3 A_1}{S_1}$$

$$\therefore \quad x = G_3 \left(\frac{S_1}{A_1+S_1}\right)\left[\frac{A_1}{S_1} + \frac{A_2 + S_2}{S_2}\right] - \frac{A_2 S_1}{S_2(A_1+S_1)} \qquad (3.21)$$

$$\therefore \text{Gear Ratio } (XR)_3 = \frac{x\left(\frac{A_1 + S_1}{S_1}\right) - \frac{G_3\ A_1}{S_1}}{x}$$

$$= \frac{G_3\left(\frac{A_2+S_2}{S_2}\right) - \frac{A_2}{S_2}}{x} \tag{3.22}$$

$$\text{and} \quad A_3 = \frac{S_3}{G_3-1} \tag{3.23}$$

3.3.1.4 Top Gear (Fig.3.7)

The top gear speed (1:1) is obtained by means of the multiplate clutch. By engaging the multiplate clutch, the sun gear of the third train sun gets connected to the inputshaft. So the sun gears of first, second and third train run at the same speed simultaneously. So the whole system moves as a solid cylinder at the engine speed which is directly fed to the propeller shaft.

3.3.2 Reverse Speed (Fig.3.8)

Brake is applied to A_4 to hold it stationary

	R_4	A_4	S_4
Step 1	+1	+1	+1
Step 2	0	-1	$\frac{A_4}{S_4}$
Result	+1	0	$1+\frac{A_4}{S_4} = w_4$

Now $A_1 = S_4$

	R_1	$A_1 (=S_4)$	S_1
Step 1	+1	+1	+1
Step 2	0	$w_4 - 1$	$(1-w_4)\frac{A_1}{S_1}$
Result	+1	w_4	$1+(1-w_4)\frac{A_1}{S_1}$

and Gear Ratio $(XR)_4 = 1 + (1 - w_4)\dfrac{A_1}{S_1}$ (3.24)

and $A_4 = S_4 (w_4 - 1)$ (3.25)

3.4 SIZE OF ANNULI

The number of teeth for the various trains are calculated, from equations derived above, as follows :

(i) I Speed

$$A_1 = S_1 \left[(XR)_1 - 1 \right] \quad (3.26)$$

(ii) II Speed

$$(XR)_2 = \frac{A_2 + S_2}{S_2\ C22} \quad (3.27)$$

where C22 $= \left[\dfrac{A_2+S_2}{A_1+S_1} \right] \dfrac{S_1}{S_2} + \dfrac{A_1}{A_1+S_1}$ (3.28)

Taking $S_1 = S_2$

$$C22 = \frac{A_2}{A_1 - S_1} + 1$$

$$A_2 = \frac{S_1\left[1-(XR)_2\right](XR)_1}{(XR)_2 - (XR)_1}$$

(iii) III Speed

$$\text{If } C_3 = G_3\left(\frac{S_1}{A_1+S_1}\right)\left[\frac{A_1}{S_1} + \frac{A_2+S_2}{S_2}\right] - \frac{A_2 S_1}{S_2(A_1+S_1)}$$

$$C_3(XR)_3 = G_3\left(\frac{A_2+S_2}{S_2}\right) - \frac{A_2}{S_2}$$

which give

$$G_3 = \frac{A_2 S_1 (XR)_3 - A_2(A_1+S_1)}{(XR)_3 A_1 S_2 + (XR)_3 S_1(A_2+S_2) - (A_1+S_1)(A_2+S_2)} \qquad (3.29)$$

and $$G_3 = 1 + \frac{S_3}{A_3}$$

or $$A_3 = \frac{S_3}{G_3 - 1} \qquad (3.30)$$

or $$= \frac{S_3\left[(XR)_3\frac{A_1}{A_2} + (XR)_1\left(1+\frac{S_2}{A_2}\right)\quad (XR)_1(XR)_3 - 1)\right]}{(XR)_3\left(1-\frac{A_1}{A_2}\right) - (XR)_1\left[1+\left(1+\frac{S_2}{A_2}\right)\left\{(XR)_1(XR)_3-1)\right\}\right]} \qquad (3.31)$$

or

$$A_3 = \frac{(XR)_3\left[(XR)_1 - 1\right]\left[(XR)_2-(XR)_1\right] + (XR)_1\left[(XR)_2-(XR)_1\right]\left[(XR)_1(XR)_3-1\right]}{(XR)_3\left[(XR)_1\left\{1-(XR)_2\right\}-\left\{(XR)_1-1\right\}\left\{(XR)_2-(XR)_1\right\}\right] - (XR)_1} S_3 \qquad (3.32)$$

(iv) Reverse Speed

$$C_4 = 1 + \frac{A_4}{S_4} \tag{3.33}$$

$$(XR)_4 = 1 + (1-C_4)\frac{A_1}{S_1} \tag{3.34}$$

or $$A_4 = \frac{\left[(XR)_4 - 1\right] S_4}{(XR)_1 - 1}$$

3.5 SUMMARY OF CONSTRAINTS USED IN PROGRAMME

1. Interference constraints

$$S_i \geqslant 16$$

2. Neighbourhood constraints

$$\sin \frac{\pi}{Q_i} \geqslant \frac{(z_2)_i + 2}{(z_1)_i + (z_3)_i}$$

3. Stress constraints or Horse Power (Torque) Constraint

$$(WB)_i \geqslant 1.35\ (WD)_i$$

or $$(S_a)_i \geqslant \frac{2T}{k^2\ m_i^3\ Y_i}$$

4. Size constraints

$$\begin{aligned} DIAA2 &\geqslant 0.95\ DIAA1 \\ &\leqslant 1.05\ DIAA1 \\ DIAA3 &\geqslant 0.85\ DIAA1 \\ &\leqslant 0.95\ DIAA1 \\ DIAA4 &\geqslant 0.95\ DIAA1 \\ &\leqslant 1.05\ DIAA1 \end{aligned}$$

5. Assembly constraints

$$\frac{A_i + S_i}{Q_i} = \text{Integer}$$

6. Velocity Ratio Constraint

$$1.02\ (XR)_i > (XR)_i > 0.98\ (XR)_i$$

7. Interference constraint for planet gears

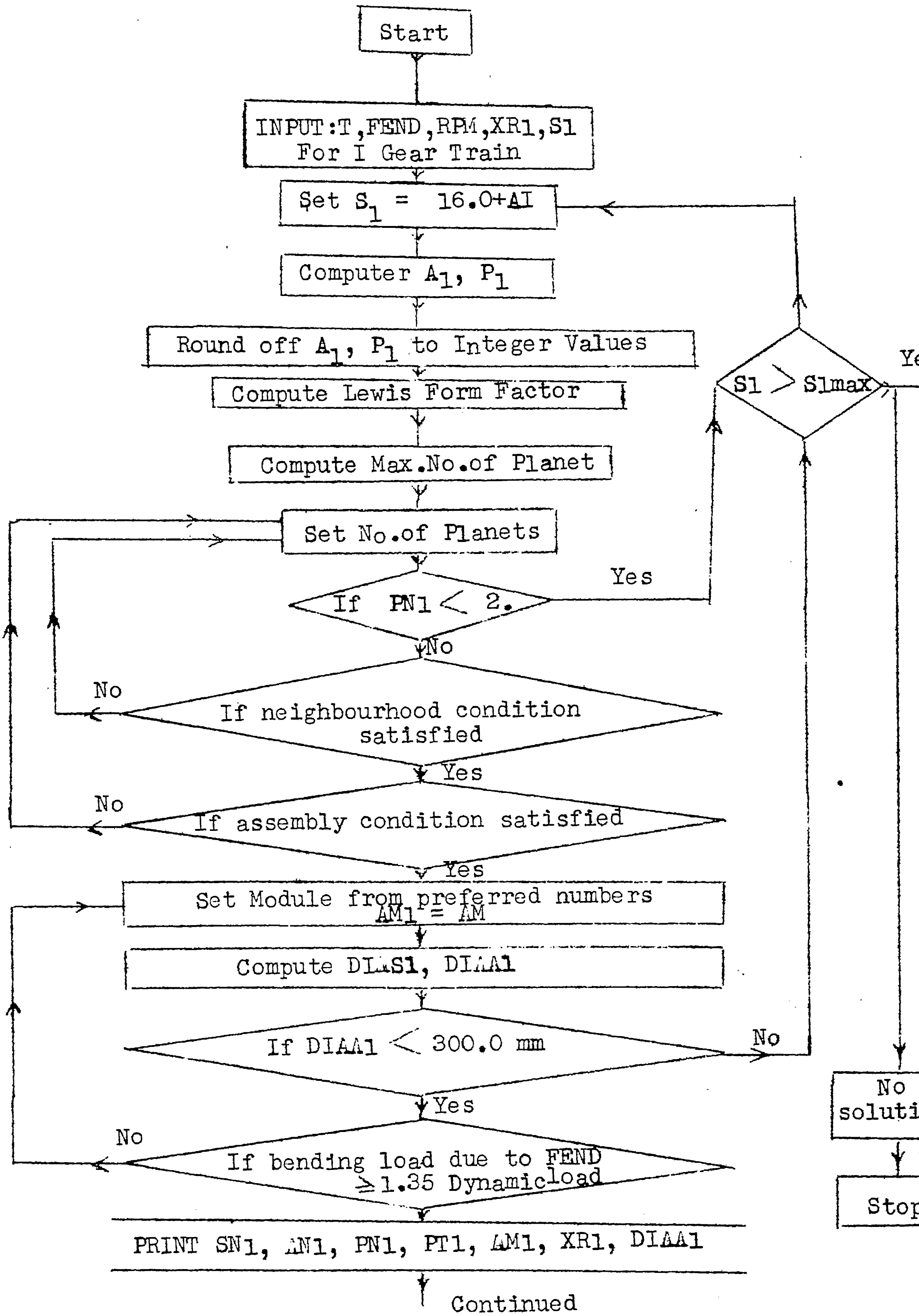
$$(PN)_i \geq 16$$

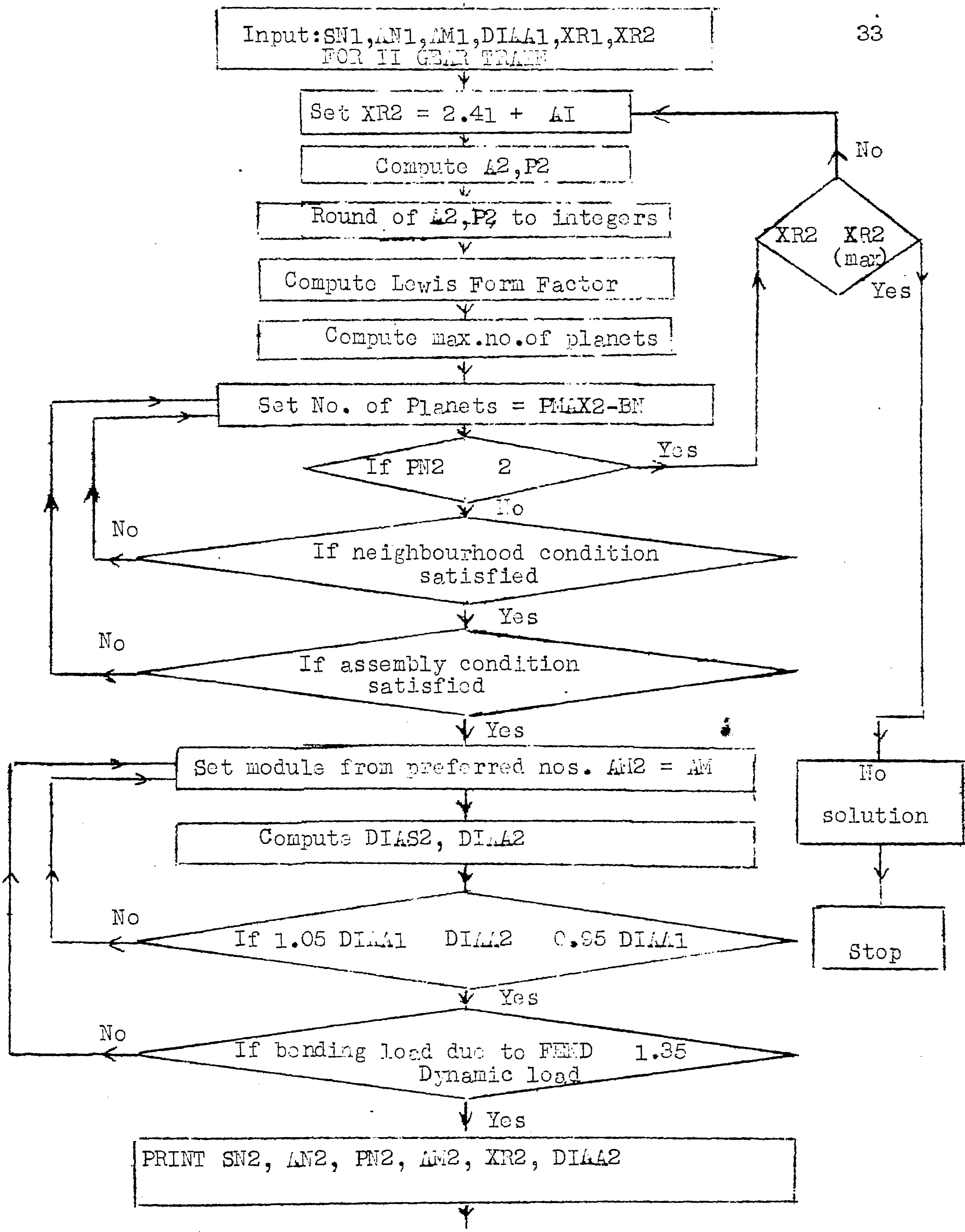
8. Module constraints

$$m_i \geq 1.0$$

3.6 COMPUTER PROGRAMMING

The programming for the design has four different stages. The output of the first gear train together with the variables of the second gear train, is the input for the third gear train. Similarly the input of the reverse gear train consists of the output of the first gear train and the velocity ratio of the reverse gear train. The flow chart given below explains the process of iteration in deciding the number of teeth of annulus and planet gears, the number of planets, module and diameter of annulus.


Start
INPUT:T,FEND,RPM,XR1,S1
For I Gear Train
Set S_1 = 16.0+AI
Computer A_1, P_1
Round off A_1, P_1 to Integer Values
Compute Lewis Form Factor
Compute Max.No.of Planet
Set No.of Planets
If PN1 < 2.
Yes
No
If neighbourhood condition satisfied
No
Yes
If assembly condition satisfied
No
Yes
S1 > S1max
Set Module from preferred numbers AM1 = AM
Compute DIAS1, DIAA1
If DIAA1 < 300.0 mm
No
Yes
If bending load due to FEND ≥ 1.35 Dynamicload
No
PRINT SN1, AN1, PN1, PT1, AM1, XR1, DIAA1
No solut
Stop
Continued



Similar flow diagram for III and reverse gear trains

3.7 RESULTS

Starting from a particular number of teeth of sun gear of first gear train, we get three sets of possible combinations of teeth of second, third and reverse gear trains. Minimum and maximum number of teeth of sun gear of first gear train were restricted to 16 and 36 respectively. Actual number of possible solutions depends on the variation of individual sizes. The actual number of solutions depending on the various tolerances on annulus diameter, is given in Table II.

In view of the innumerous number of solutions, when tolerance zone is rather slack, the results for about one percent variation on the annulus diameter of different gear trains are presented in Table I.

It is quite obvious that if one element is picked from each set, a design will follow to obtain a design, which gives the minimum volume of the gear box for any given number of teeth of the first sun gear.

The criteria of selection of the minimum size is the minimum of the function :

$$\sum_{i=1}^{4} (DIAA)_i^2 \, (AM)_i$$

(The face width being directly proportional to the module).
The results are presented in Table I.

TABLE 1

Sl. No.	No. of teeth/ module of sun of I gear train	FIRST GEAR TRAIN						SECOND GEAR TRAIN					
		XR1	No.of teeth of		PT1	DIAA1	b1	XR2	No.of teeth of			PT2	DIAA2
			A1	S1					S/M	A2	P2		
1	21	4.28	69	24	5	225	39.0	2.43	21	69	24	3	225
2	16 3.5	4.28	52	18	4	182	38.5	2.42	16 3.5	52	18	4	182
3	17	No solution											
4	18	No solution											
5	19 3.5	4.28	62	22	3	217	38.5	2.42	19 3.5	62	22	3	217
6	20 3.5	4.28	66	23	2	231	38.5	2.43	20 3.5	66	23	2	231
7	21 3.0	4.28	69	24	5	207	33.0	2.43	21 3.0	69	24	3	207
8	22 3.5	4.28	72	25	2	252	38.5	2.42	22 3.5	72	25	2	252
9	23	No solution											
10	24	No solution											
11	25	No solution											
12	26 3.0	4.28	85	30	3	255	33.0	2.42	26 3.0	85	30	3	255
13	27	No solution											
14	28 2.5	4.28	92	32	5	230	27.5	2.42	28 2.5	92	32	5	230
15	16 4.0	4.28	52	18	4	208	44.0	2.42	16 4	52	18	4	208
16	21 3.5	4.28	69	24	5	241	38.5	2.43	21 3.5	69	24	3	241

TABLE 1

THIRD GEAR TRAIN							REVERSE GEAR TRAIN							Volume of Gear Box c.c.	F
XR3	No. of Teeth of S/M	A3	P3	PT3	DIAA3	b3	XR4	No. of Teeth of S/M	A4	P4	PT4	DIAA4	b4		
1.59	24	60	18	3	200	38.0	5.97	32	68	18	5	225	39.0	5960	[illegible]
1.61	21 3.0	55	17	4	165	33.0	5.97	35 2.5	75	20	5	187.5	27.5	4850	[illegible]
No solution															
No solution															
1.57	19 3.5	56	19	3	196	38.5	5.97	33 3.0	71	19	4	213	33.0	5550	[illegible]
1.61	23 3.5	61	19	3	213.5	38.5	5.97	36 3.0	76	20	4	228	33.0	6370	
1.61	24 3.0	64	20	4	192	33.0	5.97	28 3.5	59	16	3	206.5	38.5	4630	[illegible]
1.57	22 3.5	65	21	3	227	38.5	6.01	39 3	84	22	3	252	33.0	7400	
No solution															
No solution															
No solution															
1.57	20 4.0	58	19	3	232	44.0	5.96	30 4.0	64	17	4	256	44.0	7540	
No solution															
1.58	24 3	69	23	3	207	33.0	5.96	36 3.0	76	20	4	228	33.0	4940	[illegible]
1.59	23 3.0	64	20	3	192	33.0	5.97	28 3.5	60	16	4	210	38.5	5550	[illegible]
1.59	24 3.5	64	20	3.5	224	38.5	5.97	32 3.5	68	18	5	241	38.5		

TABLE II

No. of teeth in Ist Gear	POSSIBLE NUMNER OF SOLUTIONS	
	Percentage variation of Annuli diameter of 2nd, 3rd and Reverse Trains w.r.t. Ist Train	
	1%	5%
16	3328	22248
17	-	-
18	-	-
19	252	7976
20	208	2304
21	1800	8904
22	51	2808
23	-	-
24	-	-
25	-	-
26	54	1250
27	-	
28	78	78

CHAPTER IV .

MATHEMATICAL PROGRAMMING APPROACH TO THE PROBLEM

In this chapter, the same problem is solved as a mathematical programming problem.

The general mathematical programming problem is to determine a vector $\bar{x}$ that minimizes the function $f(\bar{x})$ subject to the constraints $g_i(\bar{x}) \geq 0$, $i = 1,2,\ldots\ldots m$. For this design the vector $\bar{x}$ becomes a vector of design variables. The function $f(\bar{x})$, called the objective function, with respect to which the design is optimized as a computable function of the design variables, which expresses the qualities that make one design better than another. The constraints $g_i(\bar{x}) \geq 0$ are the design restrictions, the satisfaction of which distinguishes acceptable designs from unacceptable ones. The design which optimizes the objective function (i.e. minimizes the volume) and satisfies all the design constraints is "the best design". Because of the non-linear characteristics of the objective function and the design constraints and because of the special nature of certain constraints, the method selected is the unconstrained minimization method which uses penalty functions to handle the constraints.

4.1 MATHEMATICAL PROGRAMMING PROBLEM

4.11 The Mathematical Model

Here the mathematical model of the gear box is formulated. The term mathematical model is considered here

to mean the complete set of equations and inequalities, which are necessary for the statement of the design problem as a mathematical programming problem. This set of functions includes all of the equations and inequalities needed to describe the kinematic, geometric and dynamic requirements, including coaxiality, neighbourhood, assembly, interference, dynamic load, etc., the design function and design constraints.

4.1.2 The Objective Function

The object of the synthesis problem being considered here is to design a gear box using epicyclic gear trains which transmits the given torque and gives the designed speed ratios within 2 percent of the specified values. The values pertain to a heavy truck/bus having four forward speeds and a reverse speed.

These following twelve quantities are taken as design variables :

Modules, $m_i \equiv B_i$, $i = 1,2,3,4$

Velocity Ratios $(XR)_i \equiv B_i$, $i = 5,6,7,8$

No. of teeth on Sungears $S_i \equiv B_i$, $i = 9,10,11,12$

Any set of values for these design variables is called a 'design' even if it is absurd (i.e. gives gears which cannot be assembled) or inadequate in terms of transmission of specified energy etc.

The synthesis problem may now be stated more specifically as follows :

An epicyclic gear box is to be designed so that

i) it transmits the required torque

ii) it gives four forward speeds and one reverse speed within a variation of 2 percent.

The volume function can now be formulated as the sum of volumes of the four gear trains, the top speed being obtained directly. The volume that a gear train occupies, depends on the outside diameter of the annulus and face width of the teeth. The space occupied by the pins, arms, etc. and multiplate clutch do not directly effect the volume function as these occupy a specific space and so are neglected in the optimization problem.

$$\text{So} \quad f(\bar{x}) = \sum_{i=1}^{4} \frac{\pi}{4} D_i^2 b_i \tag{4.11}$$

The best design is the one which minimizes this total volume and satisfies all the design constraints. The design constraints are the restrictions placed on the minimum number of teeth, which can be cut without interference, the maximum number of planets, the dynamic load to remain less than the load calculated from endurance stress (or the load calculated by using velocity factor), and the number of teeth on annulus and sun gears to affect the assembly.

4.1.3 Objective Function Formulation

The object of the optimization problem is to minimize the volume of the gear box, which is given by :

$$V = \sum \text{Area of cross section of Annulus X face width}$$

$$= \sum \frac{\pi}{4} (m_i A_i)^2 \text{ X } k_i m_i \quad , i = 1,2,3,4$$

$$= \frac{\pi^2}{4} k_i \sum m_i^3 \text{ X } A_i^{2*} \tag{4.12}$$

The objective function can now be written as

$$W = \frac{\pi^2}{4} k \left[\left\{ m_1^3 (XR1 - 1)^2 S_1^2 \right\} + \left\{ m_1^3 (c_{22} S_2 XR2 - S1)^2 \right\} \right.$$

$$\left. + \left\{ m_3^3 \left(\frac{S_3}{c_{33} - 1}\right)^2 \right\} + \left\{ m_4^3 (c_{44} - 1)^2 S_4^2 \right\} \right] \tag{4.13}$$

where $c_{22} = \left(\frac{A_2 + S_2}{A_1 + S_1}\right) \frac{S_1}{S_2} + \frac{A_1}{A_1 + S_1}$

$$c_{33} = G_3\left(\frac{S_1}{A_1 + S_1}\right) \left[\frac{A_1}{S_1} + \frac{A_2 + S_2}{S_2} \right] - \frac{A_2 S_1}{S_2 (A_1 + S_1)}$$

$$G_3 = 1 + \frac{S_3}{A_3}$$

$$c_{44} = 1 + A4/S4$$

4.1.4 The Design Constraints

The design constraints or design restrictions, a satisfaction of which distinguishes acceptable designs from unacceptable designs, can be grouped into two categories. Any

---

* The expressions for the diameter of annulii A_i are already obtained in 3.11 and 3.12.

constraint which restricts the range of design variables for reasons other than the direct consideration of performance of the design is called a side constraint. A constraint which is derived from explicit consideration of performance of the design is called a behaviour constraint.

SIDE CONSTRAINTS

Since vector notation is used in the formulation of the problem, a set of side constraints must be imposed on the magnitudes of the vectors i.e.

$$S_i \geqslant 0 \qquad i = 1,2,3,4$$

BEHAVIOUR CONSTRAINTS

(a) Maximum allowable stress constraint - As discussed under 3.25 and 3.26, the dynamic stress induced in the material should be less than the allowable stress as

$$find = \frac{2T}{k\pi^2 m_i^3 Y_i S_i} \quad , \qquad i = 1,2,3,4$$

$$fall = fst \frac{3.01}{3.01+V}$$

$$m_i \equiv B(i) \qquad i = 1,2,3,4$$

$$m_1 \equiv B(1)$$

$$m_2 \equiv B(2)$$

$$m_3 \equiv B(3)$$

$$m_4 \equiv B(4)$$

$$V_i = \frac{\pi D_i N}{60}$$

$FIND_i \equiv$ find

$FALL_i \equiv$ fall

The constraints are represented as G(I) as given below :

G(1) = FIND1-FALL1

G(3) = FIND3-FALL3

G(5) = FIND4-FALL4

As these calculations are for sun gears, and sun gear of first and second train is same, the constraint is expressed only for the first gear train sun wheel only.

Alternatively, using Buckingham equation for dynamic load,

G(1) = WB1 - 1.35 WD1

G(3) = WB3 - 1.35 WD3

G(5) = WB4 - 1.35 WD4

(b) Neighbourhood Constraint

$$CI_i = \frac{(P_2)_i + 2}{A_i + S_i} \qquad i = 1,3,4$$

G(2) = CI1 - 3.0

G(4) = CI3 - 3.0

G(6) = CI4 - 3.0

(c) Annulii Size Constraint

The diameter of second, third and reverse train annulii is restricted with respect to the diameter of the first train annulus as

$$DIF1 = ABS\left(\frac{D1-D4}{D1}\right)$$

$$DIF2 = ABS\left(\frac{D1-D2}{D1}\right)$$

$$G(7) = 0.85\ D1 - D3$$

$$G(8) = 0.15 - DIF1$$

$$G(9) = 0.15 - DIF2$$

(d) Module Constraint

The minimum value of modules used in any of the train should be greater than 1 i.e.

$$B(i) = m_i \geqslant 1$$

Or

$$G(10) = B(1) - 1.0$$

$$G(11) = B(2) - 1.0$$

$$G(12) = B(3) - 1.0$$

$$G(13) = B(4) - 1.0$$

(e) Velocity Ratio Constraint

The velocity ratio is allowed a variation of 2 per cent from the prescribed values. These are expressed as follows :

$$(VR)_i \equiv B(i) \qquad i = 5,6,7,8$$

Or

$$G(14) = B(5) - 4.195$$

$$G(15) = 4.36 - B(5)$$

$$G(16) = B(6) - 2.3814$$

$$G(17) = 2.4786 - B(6)$$

$$G(18) = B(7) - 1.5582$$

$$G(19) = 1.6218 - B(7)$$

$$G(20) = B(8) - 5.8306$$

$$G(21) = 6.1094 - B(8)$$

(f) Interference Constraint

To avoid interference, there is a limitation to the number of teeth of sun gears (3.5) i.e.

$$S_i - 16 \geq 0$$

Representing $S1 \equiv B(9)$

$S2 \equiv B(10)$

$S3 \equiv B(11)$

$S4 \equiv B(12)$

These constraints are formulated below :

$$G(22) = B(9) - 16.0$$

$$G(23) = B(10) - 16.0$$

$$G(24) = B(11) - 16.0$$

$$G(25) = B(12) - 16.0$$

(g) Assembly constraint

The assembly constraint is not used in the programme, because it cannot be used as such. This is given by

$$\frac{A_i + S_i}{Q} = \text{Integer} \qquad i = 1,2,3,4$$

All these quantities are integers.

4.2 SOLUTION SCHEME

The method chosen here is the unconstrained minimization : Variable Metric Method of Fletcher and Powell[33].

The number of teeth of gears and the number of planets must be integers. The modules must be selected from the preferred numbers. The assembly constraint consists of all integers. These requirements make the problem as Non-linear with mixed integer and real variables.

The only way to solve this problem is to float the variables and obtain the minimas. Then these variables are given the nearest integer values or preferred values in the case of modules. These modified values may or may not lead to the minimas. As this method is used only as a check for the previously obtained results, the values of variables are compared with the values obtained from optimum solutions of each set.

The scheme of solution of this method is briefly discussed below :

It requires only first order partial derivatives of the function with respect to the design variables. The steps required for the minimization of a function are as follows :

(i) To select a starting point $\bar{x}^o$ and $[H^o]$, where $[H^o]$ is any positive definite matrix of the order nxn, where n is the number of design variables. In general to start with $[H^o]$ is taken to be an identity matrix.

(ii) To find the gradient at $\bar{x}^o$, $\nabla f(\bar{x}^o)$. For the i^{th} iteration $\bar{x}^i$, the gradient is $\nabla f(\bar{x}^i)$ and the positive definite matrix is $[H^i]$.

(iii) Find the next feasible point as follows

$$\text{Let } \bar{S}^i = -[H^i] \nabla f(\bar{x}^i) \tag{4.23}$$

$$\text{and } \bar{\sigma}^i = \lambda^i \bar{S}^i \tag{4.24}$$

$$\bar{x}^{i+1} = \bar{x}^i + \bar{\sigma}^i \tag{4.25}$$

where λ^i is a scalar and is greater than zero, which is chosen such that it minimize f along $\bar{S}^i$ starting at $\bar{x}^i$.

(iv) Evaluate $\bar{y}^i = \nabla f(\bar{x}^{i+1}) - \nabla f(\bar{x}^i)$ (4.26)

(v) To solve the following matrix at $(i+1)^{th}$ iteration :

$$[H^{i+1}] = [H^i] + \frac{(\bar{\sigma}^i)(\bar{\sigma}^i)^T}{(\sigma^i)^T \bar{y}^i} - \frac{[H^i](\bar{y}^i)(\bar{y}^i)^T[H^i]}{(y^i)^T [H^i] (\bar{y}^i)} \tag{4.27}$$

(vi) To begin the next iteration from step (iii)

(vii) To stop algerithm when convergence criterion is satisfied.

4.2.1 CONVERSION TO UNCONSTRAINED MINIMIZATION

The general mathematical problem is to determine a vector $\bar{x}$ that minimizes the function $f(\bar{x})$ subject to the constraints

$$g_i(\bar{x}) \geq 0 \qquad i = 1,2,\ldots\ldots,m$$

Fiacco and Mc Cormick[33] have developed an algorithm for transforming this mathematical programming problem with constraints into a sequence of unconstrained minimization problem. The procedure is based on the minimization of a new function called the Penalty function defined as :

$$P\,(\bar{x},r) = f(\bar{x}) + r \sum_{i=1}^{m} \frac{1}{g_i(\bar{x})}$$

over a strictly monotonic decreasing sequence of r-values, rk. The conditions on $f(x)$ and $g_i(x)$ are :

(a) $f(\bar{x})$ and $g_i(\bar{x})$, $i = 1,\ldots\ldots,m$ are convex functions and

(b) $P\ (\bar{x},r)$ is strictly convex in the interior of the constraint set for every $r > 0$.

Then, the optimal solution to the constrained problem approaches the global minimum of the constrained problem as r approaches zero.

For $r_1 > 0$ and a point $\bar{x}^{\,o}$ strictly within the constrained set that is $g_i(\bar{x}^{\,o})$ 0, $i = 1,\ldots\ldots,m$, it is expected that a minimum of $P\,(\bar{x},r)$ exists inside the constrained set, since on the boundary of the constrained set some $g_i(x)$ must be equal to zero and $P_f(\bar{x},r_1)$ depends on the value of r_1, and is denoted by $\bar{x}(r_1)$. The point $\bar{x}(r_1)$ lies

value of r_1, and is denoted by $\bar{x}(r_1)$. The point $\bar{x}(r_1)$ lies inside the constrained set. By reducing the value of r, the influence of summation term, which penalizes the closeness to the constrained boundaries, is reduced, and in the minimizing $P_f(\bar{x},r)$, more emphasis is placed upon decreasing f(x). If the minimization process is repeated for decreasing sequence of r values, each minimum point $\bar{x}(r_k)$ can move closer to the boundaries of the constraint set if it is profitable in terms of reducing $f(\bar{x})$. This method is particularly attractive for the problems which have non-linear constraints, since it approaches the solution from points inside the constraint set and thus avoids the difficult movement along the non-linear boundary of the set.

If

(a) the interior of the constraint set is nonempty

(b) the function $f(\bar{x})$ and $-g_i(\bar{x})$, $i = 1,2,\ldots\ldots,m$, are continuously differentiable

(c) the set of points in the constraint set for which $f(\bar{x}) \leqslant V_0$ is bounded for every finite V_0, and

(d) the function $f(\bar{x})$ is bounded below for $\bar{x}$ in the constraint set, then

there is optimal solution to the constrained problem as the value of r approaches zero.

Now to apply the Fiacco and Mc Cormick algorithm to this problem, the Penalty function is formed for the gear box

volume :

$$P_f(\bar{x},r) = V + r \sum \frac{1}{g_i(x)}$$

$$= V + r \left[\frac{1}{FIND1-FALL1} + \frac{1}{CI1-3.0} + \frac{1}{FIND2-FALL2} + \frac{1}{CI3-3.0} + \ldots\ldots \right]$$

The conditions mentioned above in this section, if they are not satisfied, it is possible that the algorithm may not lead to a global minimum. In such a case one has to start from different feasible points and obtain the minimum of the minima. If on the other hand minimum obtained from different starting points is same, one can conclude that the minimum obtained is also global.

4.2.2 Computational Aspect of Penalty Function

Following steps are involved in this technique :

(i) To choose an initial feasible point $\bar{x}_0$.

(ii) To select r_1, the initial value of r. The selection of initial value of r is quite important. Initially it would seem advantageous to select $r_1 = \varepsilon > 0$, where ε is selected as small as possible, without loosing significance due to round off errors, etc., but it effects the rate of convergence.

Below is given a method of choosing r_1, which is usually adopted.

Since a necessary condition for $P_f(\bar{x},r)$ to be a minimum is the vanishing of the first partial derivative a natural choice of r_1 would be given by r that minimizes the magnitude of the square of the gradient of $P_f(\bar{x},r)$ at $\bar{x}^o$, i.e.

$$\min_r \left| P_f(\bar{x}^o,r) \right|^2 = \min_r \left| \nabla f(\bar{x}) - r \sum_{i=1}^{m} \nabla g_i(\bar{x}^o) \middle| g_i^2(\bar{x}^o) \right|^2 \tag{4.29}$$

For simplicity, let $P(\bar{x}) = \sum_{i=1}^{m} \frac{1}{g_i(\bar{x})}$, then r_1 is given by,

$$r_1 = - \nabla f(\bar{x}^o)^T \nabla p(\bar{x}^o) / \left| p(\bar{x}^o) \right|^2 \tag{4.30}$$

Equation (4.29) is obtained by differentiating (4.28) with respect to r and equating the resulting expression to zero. Equation (4.30) can be used if $r_1 > 0$ which is obtained if

$$\nabla f(\bar{x}^o)^T \nabla p(\bar{x}^o) < 0 \text{ . If } \nabla f(\bar{x}^o)^T \nabla p(\bar{x}^o) \geqslant 0 \tag{4.31}$$

gives $r_1 \leqslant 0$, but in this case $f(\bar{x})$ can be decreased at its maximum spatial rate, without increasing the $\sum$ term of the P-function, by proceeding along $-\nabla f(\bar{x}^o)$. In this case, therefore, taking a sequence of steps of given length down the gradient f and recomputing (4.30) at each new point until a positive r_1 is obtained, is an efficient way to proceed.

(iii) To determine minimum of $P_f(\bar{x},r_k)$, r_k being the value of r at the k^{th} iteration.

(iv) To terminate solution of the above step if the convergence criterion is satisfied. A convergence criterion is when $|\nabla P_f(\bar{x})| < \varepsilon$, where $\varepsilon > 0$, but a small specified quantity.

(v) To select $r_{k+1} = C_1 r_k$ where $0 < C_1 < 1$ and to continue the procedure from step 3 for the new value of r.

(vi) To stop algorithm, when the final convergence criteron is satisfied. A final convergence criterion is one when

$r \sum_{i=1}^{m} \frac{1}{g_i(\bar{x})} < \delta$ where $\delta > 0$ is a very small quantity.

For the Fletcher & Powels method, the gradient of the objective function and constraints at $\bar{x}$ are required. These are obtained by finite differences method as :

<u>4.2.3 The Gradient</u>

The gradient of a n-dimensional function is defined as :

$$\bar{G}(\bar{x}) \equiv \nabla F(\bar{x}) \equiv \left(\frac{\partial F}{\partial x_1}, \frac{\partial F}{\partial x_2}, \ldots\ldots \frac{\partial F}{\partial x_n}\right) \ldots\ldots \qquad (4.32)$$

or it may be expressed as

$$\nabla \phi(x) = \left[\left(\frac{\partial F}{\partial x_1} + r \sum_{i=1}^{m} \frac{1}{g_i^2} \cdot \frac{\partial g_i}{\partial x_1}\right), \left(\frac{\partial f}{\partial x_2} + r \sum_{i=1}^{m} \frac{1}{g_i^2} \cdot \frac{\partial g_i}{\partial x_2}\right), \ldots \right.$$

$$\left. \ldots, \left(\frac{\partial f}{\partial x_n} + r \sum_{i=1}^{m} \frac{1}{g_i^2} \frac{\partial g_i}{\partial x_n}\right)\right]^T \qquad (4.33)$$

Thus the gradient of the interior penalty function for a fixed value of r is

$$\nabla \phi\,(\bar{x}) = \left(\frac{\partial \phi}{\partial x_1}, \frac{\partial \phi}{\partial x_2}, \ldots\ldots \frac{\partial \phi}{\partial x_n}\right)^T \tag{4.33}$$

or it may be expressed as

$$\nabla \phi(x) = \Big[\Big(\frac{\partial f}{\partial x_1} + r \sum_{i=1}^{m} \frac{1}{g_i^2} \cdot \frac{\partial g_i}{\partial x_1}\Big), \Big(\frac{\partial f}{\partial x_2} + r \sum_{i=1}^{m} \frac{1}{g_i^2} \cdot \frac{\partial g_i}{\partial x_2}\Big), \ldots\ldots \Big(\frac{\partial f}{\partial x_n} + r \sum_{i=1}^{m} \frac{1}{g_i^2} \cdot \frac{\partial g_i}{\partial x_n}\Big)^T \tag{4.34}$$

In most design problems of even moderate complexity, it is simply impractical to derive analytical expressions for all of the $\frac{\partial f}{x}$ and $\frac{\partial g}{x}$ terms. Therefore it is necessary to resort to finite difference derivations.

The difference between the accuracy of eq.(4.33) and that of eq.(4.34) is due to the differences among the functions ϕ, f and g. When $\bar{x}$ is near a constraint surface and r is small, ϕ becomes a very distorted function while f and g usually remain well behaved. Examining the Taylor series expansion of a function of one variable about the point x_o.

$$F(x_1) = F(x_o) + \left(\frac{\partial F}{\partial x}\right)\Big|_{x_o} (x_1 - x_o) + \frac{1}{2}\left(\frac{\partial^2 F}{\partial x^2}\right)\Big|_{x_o} \cdot (x_1 - x_o)^2 + \cdots \tag{4.35}$$

Solving for the first derivative yields

$$\left(\frac{\partial F}{\partial x}\right)\Big|_{x_o} = \frac{F(x_1)-F(x_o)-\frac{1}{2}\left(\frac{\partial^2 F}{\partial x^2}\right)\Big|_{x_o}(x_1-x_o)-\cdots}{(x_1-x_o)} \tag{4.36}$$

If the second and higher order derivatives can be neglected, then

$$\left(\frac{\partial F}{\partial x}\right)\Big|_{x_o} \simeq \frac{F(x_1)-F(x_o)}{x_1-x_o} \tag{4.37}$$

is a good approximation to eq.(4.36).

The gradient obtained from eq.(4.37) and (4.34) is accurate enough for use in the variable metric method.

<u>4.2.4. Determination of Step Length</u>

If $\bar{x}_o$ is the initial feasible point, the move to reach $\bar{x}^1$ is given by :

$$\bar{x}^1 = \bar{x}^0 - \lambda^o [\bar{H}^1]\, \bar{x}^o \nabla P_f\, (\bar{x}^o,\, r) \tag{4.38}$$

where $P_f\,(\bar{x}^o, r)$ is the gradient vector and $\lambda^o > 0$ is the step length chosen to minimize P_f along $-[\bar{H}^1]\,\bar{x}^o P_f(\bar{x}^o)$, so that $\bar{x}^1$ is a feasible point. λ^o is determined as follows :

First an upper limit λ^o is determined so that none of the non-negativity restrictions are violated. Let the vector $-[\bar{H}^1]\,\bar{x}^o \nabla P_f(\bar{x}^o,\, r)$ is denoted by

$$\bar{d}_o = \begin{bmatrix} d_1 \\ d \end{bmatrix} = -[\bar{H}^1]\,\bar{x}^o \nabla P_f(\bar{x}^o, r) \tag{4.39}$$

$$\text{Let } \beta = \min_{d_i < 0} \left[-\frac{x_{io}}{d_{io}}\right], \quad i = 1,\ldots\ldots.12 \tag{4.40}$$

$$\text{where } \bar{x}^o = \begin{bmatrix} x_1 \\ x_{12} \end{bmatrix} \quad . \tag{4.41}$$

If $\lambda < \beta$ then none of the non-negativity restrictions are violated.

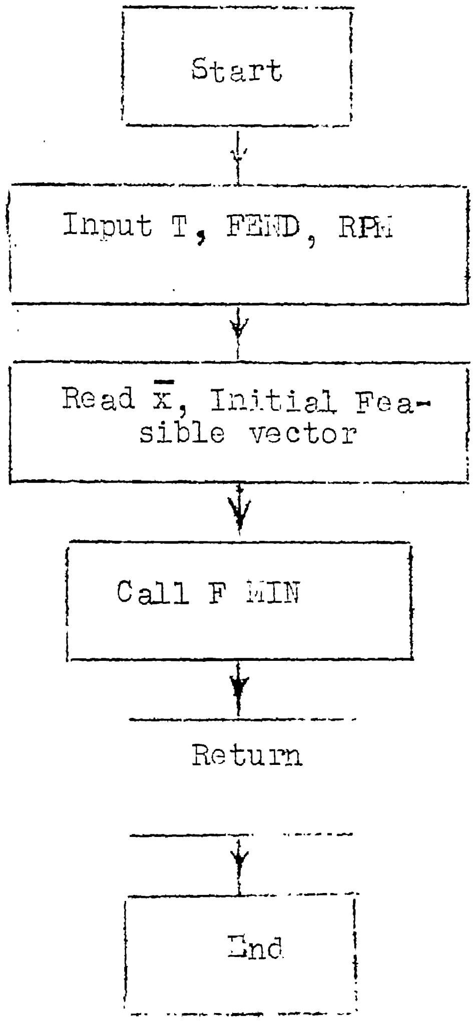
First this β is divided into two equal parts and more gradually down each time taking a step length equal to 0.5β. At each new point it is checked that along this direction, whether the P-function has decreased and the new point is the feasible point.

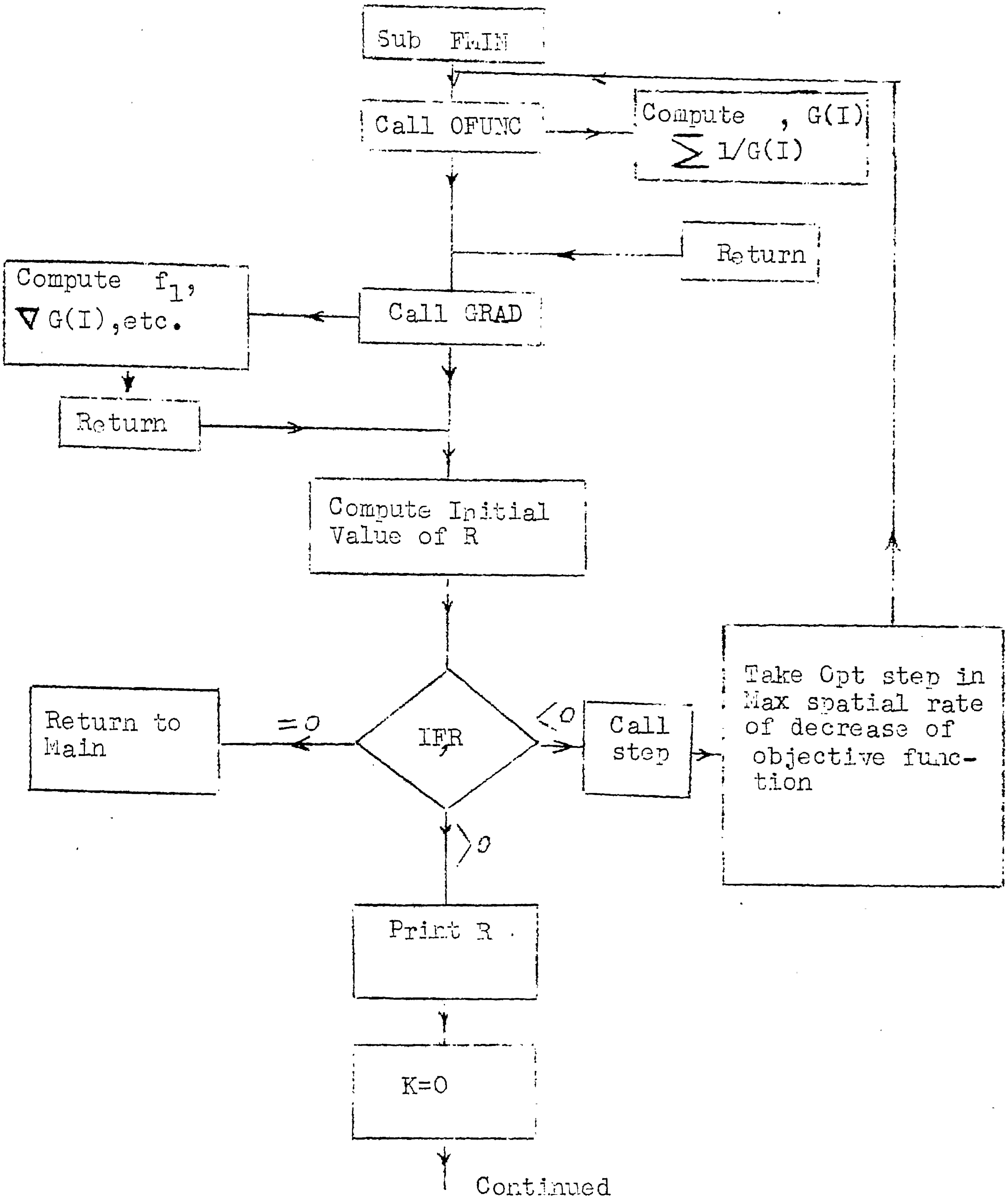
If all the constraints are satisfied and the P-function has decreased from its previous value in that direction, then we further move in the same direction. If any of the above conditions are not satisfied, the step size is reduced further to half. If the step size becomes a small fraction of β, consistent with the required precision, say one thousandth of β, the move along that direction is stopped. Thus λ is determined by this numerical search.

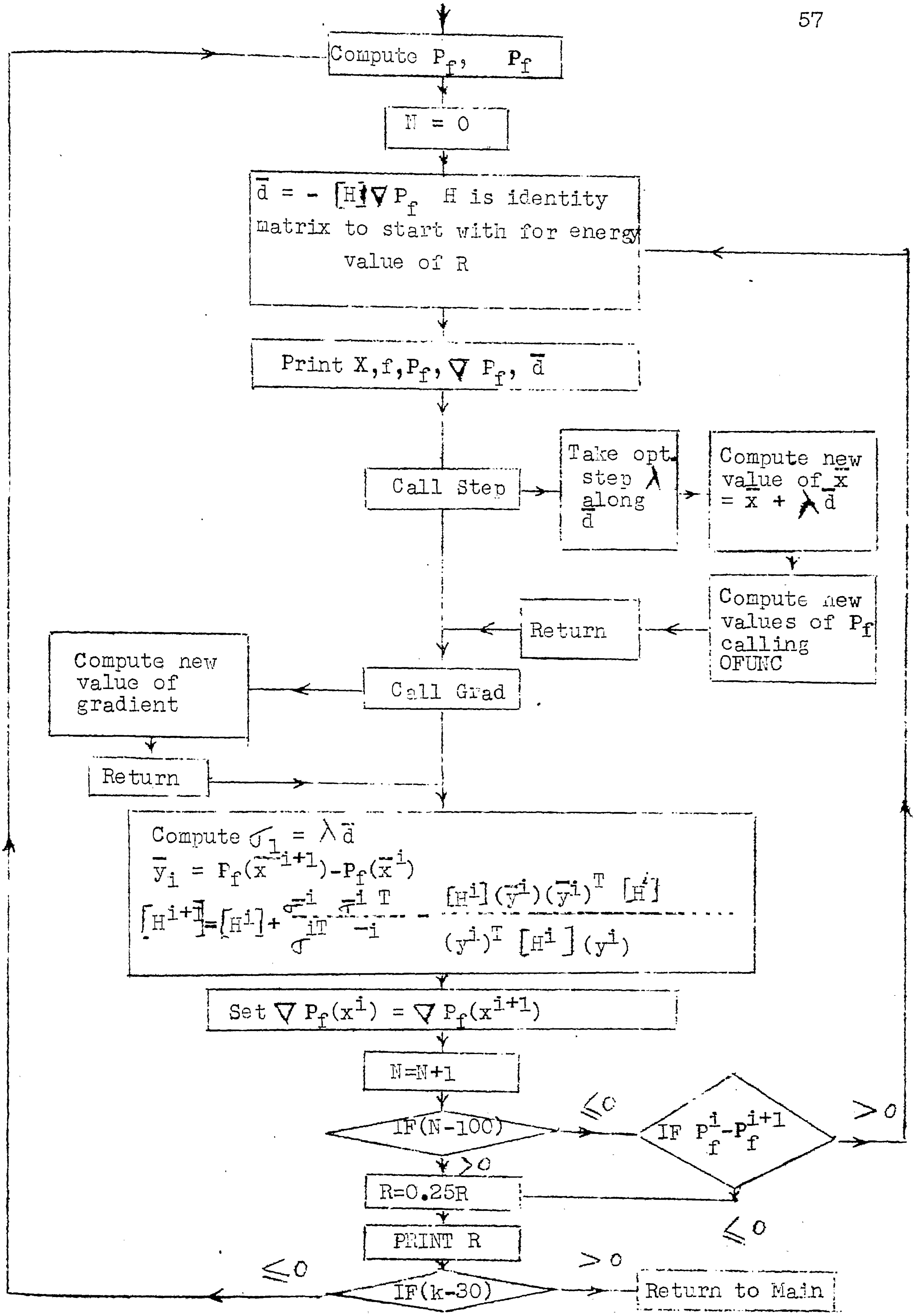
After getting the point $\bar{x}^1$ from x^o, further $\bar{x}^2$, $\bar{x}^3$,.... are obtained by application of the same procedure. The P-function is approximately minimized for a particular value of r at a point $\bar{x}^{k+1}$ when $P_f(\bar{x}^{k+1}) - P_f(\bar{x}^k) = 0$
The r value is decreased by quarter and half and the minimization is continued for a decreasing sequence of r. The algorithm is stopped when the change in $\bar{x}$ minimizing the P-function for two successive values of r is less than 0.00001 and the difference in the value of objective function and P-function is also less than 0.0005.

FLOW DIAGRAM FOR PENALTY FUNCTION APPROACH

MAIN PROGRAM

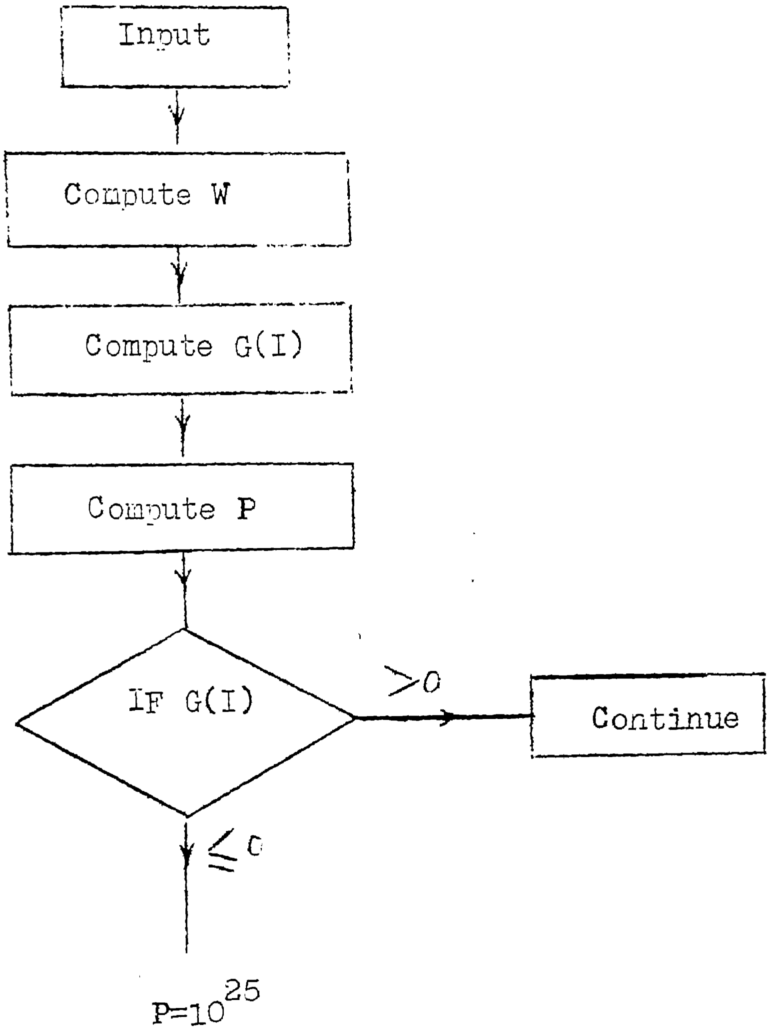



Sub FMIN
Call OFUNC
Compute , G(I)
∑ 1/G(I)
Return
Compute f1,
∇ G(I),etc.
Call GRAD
Return
Compute Initial Value of R
Return to Main
=0
IFR
<0
Call step
Take Opt step in Max spatial rate of decrease of objective function
>0
Print R
K=0
Continued

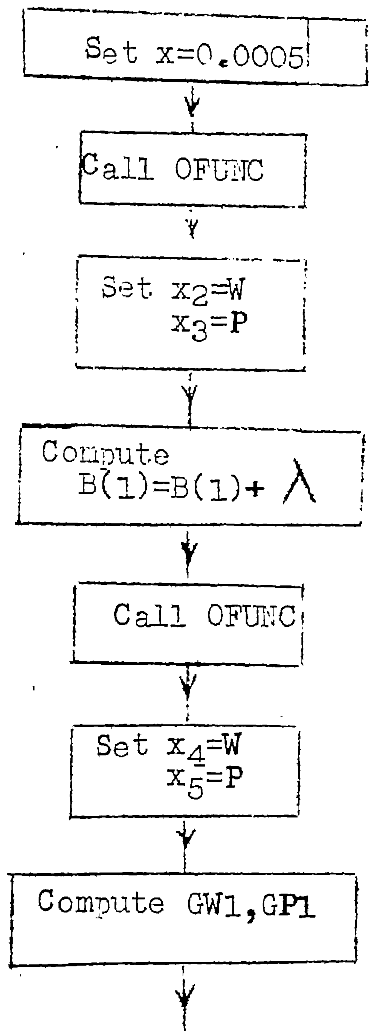


Compute P_f, ∇P_f
N = 0
$\bar{d} = -[H]\nabla P_f$ H is identity matrix to start with for energy value of R
Print X, f, P_f, ∇P_f, $\bar{d}$
Call Step
Take opt. step λ along $\bar{d}$
Compute new value of $\bar{x} = \bar{x} + \lambda\bar{d}$
Compute new values of P_f calling OFUNC
Return
Call Grad
Compute new value of gradient
Return
Compute $\sigma_1 = \lambda\bar{d}$
$\bar{y}_i = P_f(\bar{x}^{i+1}) - P_f(\bar{x}^i)$
$[H^{i+1}] = [H^i] + \frac{\bar{\sigma}^i \bar{\sigma}^{iT}}{\sigma^{iT}\bar{y}^i} - \frac{[H^i](\bar{y}^i)(\bar{y}^i)^T[H^i]}{(y^i)^T[H^i](y^i)}$
Set $\nabla P_f(x^i) = \nabla P_f(x^{i+1})$
N=N+1
IF(N-100)
≤ 0
> 0
IF $P_f^i - P_f^{i+1}$
> 0
≤ 0
R=0.25R
PRINT R
IF(k-30)
≤ 0
> 0
Return to Main

SUB ROUTINE OFUNC

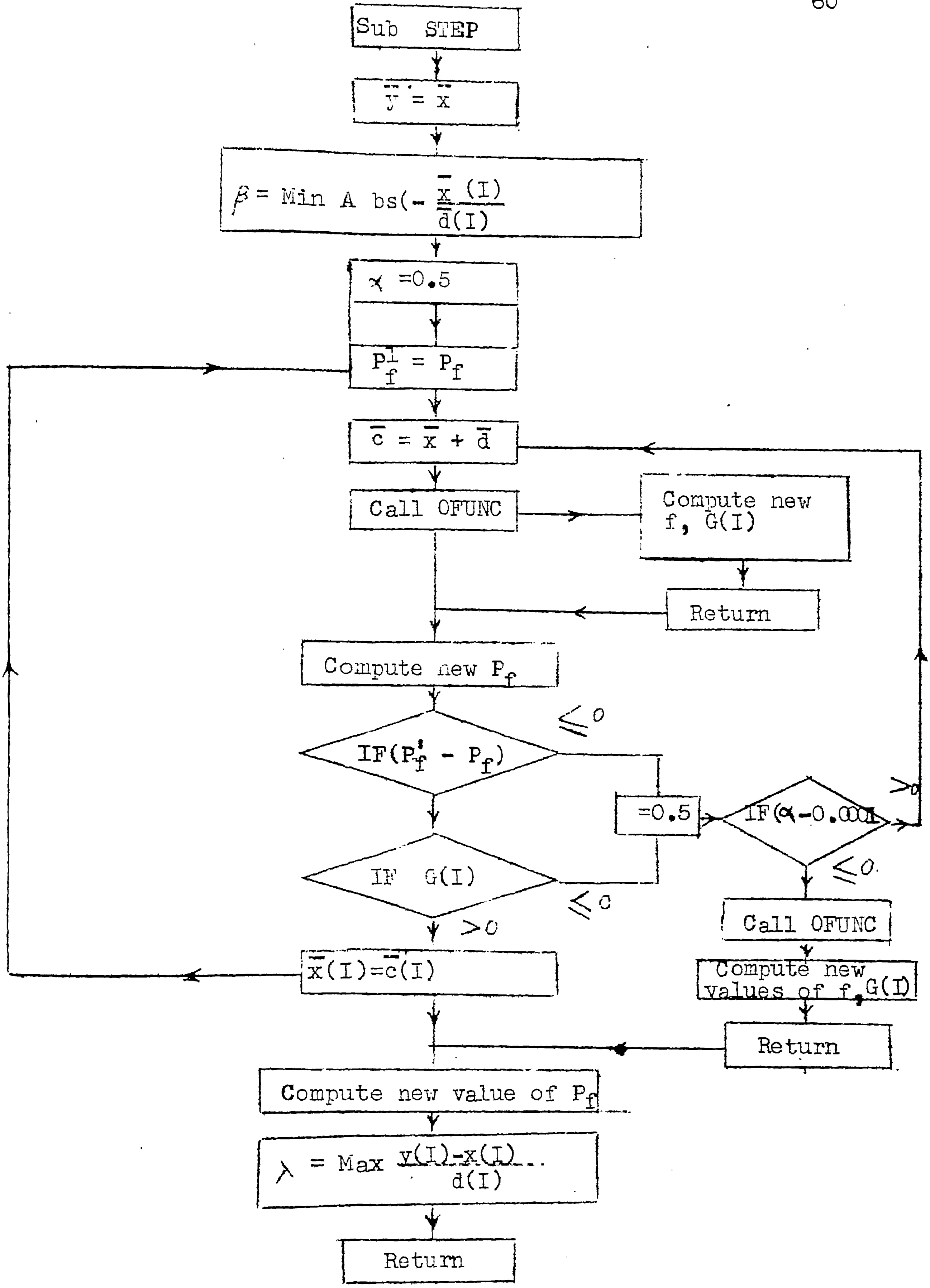


SUB ROUTINE GRAD



Repeat 11 times

SUB ROUTINE STEP



4.3 RESULTS

The results obtained from penalty function approach are listed below. Each set of variables found, correspond to a feasible solution.

Sl. No.		No. of teeth of Sun Gears				Modules				Velocity Ratios			
		I	II	III	Reverse	I	II	III	Reverse	I	II	III	Reverse
	Initial sol.	16	16	21	35	3.5	3.5	3.0	2.5	4.28	2.42	1.61	5.97
2	Sol.obtained	16	16.1	20.3	34.4	3.5	3.5	2.97	2.5	4.20	2.48	1.58	5.96
	Modified sol.	16	16	21	35	3.5	3.5	3.0	2.5	4.20	2.48	1.58	5.96
	Initial sol.	19	19	19	33	3.5	3.5	3.5	3.0	4.28	2.42	1.57	5.97
5	Sol.obtained	19	19	19	33	3.5	3.5	3.49	3.0	4.23	2.48	1.61	5.97
	Modified sol.	19	19	19	33	3.5	3.5	3.49	3.0	4.23	2.48	1.61	5.97
	Initial sol.	21	21	24	28	3.0	3.0	3.0	3.5	4.28	2.43	1.61	5.98
7	Sol.obtained	21	21	24	28	3.0	3.0	3.0	3.5	4.26	2.45	1.60	5.98
	Modified sol.	21	21	24	28	3.0	3.0	3.0	3.5	4.26	2.45	1.60	5.98
	Initial sol.	28	28	24	36	2.5	2.5	3.0	3.0	4.28	2.42	1.58	5.96
14	Sol.obtained	28	28	24	36	2.5	2.5	3.0	3.0	4.28	2.42	1.58	5.95
	Modified sol.	28	28	24	36	2.5	2.5	3.0	3.0	4.28	2.42	1.58	5.95

CHAPTER V

DISCUSSION OF RESULTS

The results of the computer aided design are given in Table I of Chapter III. By taking different number of teeth on the sun gear of first gear train, the second, third and reverse gear sizes are obtained by using iterative procedure. From the various solutions obtained corresponding to each of 16-36 number of teeth of sun gear of first gear train, the minimum volume solution has been picked up from each of the subsets. From these minimum solutions, the minimum most has been selected as the best design.

While formulating the design procedure of gears, Barth equation and Buckingham's equation were used separately. The Barth equation, being less conservative, leads to much lower values of volume as compared to the Buckingham's equation. The lowest being approximately 52% of the existing one. The number of solutions also were much more than those obtained from Buckingham's equation for the same tolerance on the diameter of annuli of the gear trains. In the present case, the design has been based upon the Buckingham's equation, because it predicts nicely the value of the dynamic load and it has given more satisfactory designs in practice.

In order to check that the optimum solution by the search technique is actually the minimum most, the penalty function approach is tried. Some of the minimum solutions of the search technique were fed as initial feasible solutions. After some iterations it leads to almost the same solution, which shows that the initial solution was near to the local minima. The results in different iterations also show that the slight variation in the variables violates one or the other constraint indicating that the feasible region is very small and the penalty function is giving the local minima. In other words, the possible design space, in the case of epicyclic gear drives is not a continuous one. Bounded by constraint surfaces, it constitutes rather a number of discrete patches in the design surface (mainly because of tight constraints on the number of teeth).

The optimum solution (Design No. 7) obtained occupies about 78 percent of the existing volume of the gear box which shows that there will be proportionate saving of material, space occupied and all other dependent factors. From the various solutions obtained, one of the solutions (Design No.16) is almost exactly the same as the existing one which justifies the selection of material, strength and the procedure adopted.

APPENDIX A

DESCRIPTION OF SOME CRITICAL PARTS AND PROBLEMS

The gear box provides a means of overcoming the initial inertia of the automobile and assists the engine while it is in motion by making the minimum demand on its power, when extra resistance is encountered. The gear box also provides a means of uncoupling the engine from the remainder of the transmission when used in conjunction with the clutch.

MATERIALS

The pressure on the teeth of gears is extremely high and the material employed in the manufacture of the gear must possess toughness and hard wearing qualities. Usually a high nickel content is called for and such materials as nickel-chrome molybdenum steel and nickel molybdenum steel are used. They are heat treated in the course of manufacture of the gear. The same material is also used for the shafts. This also facilitates pinions to be integral with the shaft. In the formation of casing and extensions, cast iron is commonly used, but some manufacturers favour aluminium alloys on the score of lightness and die castings are fairly common. Bushes are of phosphor bronze or lead bronze.

The Wilson Gear Box forms the basis of all the different makes, as they only vary in detail such as selector and toggle action, oil pump or lubrication and design of

clutch for top speed. All types retain its outstanding features i.e. employ servobrakes in such a manner as to cancel out the braking strains on the annuli and the compounding of different epicyclic trains so as to reduce speed and stresses of planet gears.

DESCRIPTION OF SOME PARTS

Brakes and clutches

The important advantage which epicyclic gears have over layshaft gears is the ability to change from one ratio to another, without loss of torque transmission. This is achieved by suitable clutches or brakes associated with the reaction members, one of which is released as another is engaged. This is called 'Hot Shift'.

Since a torque balance must exist when transmitting power, it will be apparent that the torque in the brake or clutch holding a reaction member is the difference between input and output torque. Thus for large gear steps, the reaction torque is relatively high and it is common practice to use a self-energizing brake. For lower ratios where the reaction torque is small, plate or cone friction clutches are more commonly employed. The potential speed of a reaction member is inversely proportional to the torque required to hold it stationary i.e. a reaction member, which by virtue of the gearing, would run at high speeds with the output member stalled, requires only a small torque to hold it

stationary. The actual energy which is absorbed (and dissipated as heat) in a friction clutch or brake during a shift is related to the torque transmitted and the ratio step and is independent of the potential speed difference.

Bands are useful for holding high torque reaction members as, for instance, with low forward and reverse drive. The self-wrapping servo effect of the band can be used to advantage in such cases, but band brakes of this type may be difficult to control to give smooth engagements and may not readily release on disengagement.

Band brakes are usually applied by hydraulic servo-cylinders acting directly, or through levers, on one end of the band, while the other end is anchored to the casing. Care should be taken to avoid high loadings being applied to the casing at the servo-cylinder and the band anchorages.

Plate clutches used are usually of the multi-disc type. These give a large area of surface and, hence, energy capacity. These clutches are engaged by axial forces which are derived from springs or hydraulic cylinders. For dynamic engagements, smoothness is improved by modulating the hydraulic pressure. To ensure quick disengagements and so avoid torque reversals, it is usual to separate the clutch plates by light springs.

Cone clutches are not widely used with epicyclic gear trains because the energy which a cone clutch can absorb during an engagement is limited as compared with the

multidisc clutch, but other important advantages are available - smaller space, particularly in axial length, lower cost, lower clamping load due to wedge section of the cone and lower drag when disengaged.

Bearings

In considering the bearings for the elements of an epicyclic train, the requirement for supporting the main elements is simplified by the inherent balance obtained by the use of two or more planets. Concentricity is important to ensure equal load sharing at each tooth contact point. In the absence of perfect concentricity it has been found that considerable freedom of one or two of the elements can permit automatic centering under load.

The most difficult bearing problem in an epicyclic gear train is that of the planet pinion bearings. The problem can be more apparent in an overdrive unit. At high road speed, the transmission is normally 1:1 ratio with gear trains rotating as a whole, but with an overdrive the planets are working continuously over very long periods. The planets, as well as turning on their own axes, orbit about the axis of the carrier, thus developing very appreciable centrifugal force at high carrier speeds. For this reason planet bores must be as large as possible to reduce weight, although from a consideration of tooth bending stresses, the amount of metal below the tooth root should not be less than the full tooth depth.

The needle bearing has come to be widely used for the planets of epicyclic trains and although some lubrication is essential, the necessity for full pressure oil supply does not arise as with plain bearings.

Making the gear bore as large as possible permits the use of a pin of minimum deflection. This is important with needle bearings since any appreciable deflection causes locally high loading on rollers. The distribution of loading on the bearings, even of simple planets, is not uniform owing to the effect of thrust couples and centrifugal force, it is therefore, usual where space permits, to use two separate sets of rollers suitably spaced apart.

Moreover, the centrifugal force acting on the needles becomes a problem as a result of the tooth loading and the centrifugal force moving towards the underside of planets. The needles which are loaded on the underside roll as they should, but where the maximum clearance exists at the point opposite the resultant load, the needles tend to accumulate and lose their rotational speed. When passing into the load area again they skid as they are accelerated to full speed.

If there is excessive diametral clearance, loose rollers can run out of parallel with the planet pin axis, causing further skidding and heat generation. For this reason, caged roller bearings are sometimes used, where a smaller number of rollers is employed, with a light steel or aluminum

cage to hold them parallel with the pins. Such caged bearings themselves introduce fresh problems, by virtue of the centrifugal loading on the cage itself.

All these problems are associated with high speed rather than with high load. Appreciable advantages can be obtained by improving the lubrication over that which obtain with simple oil grooves, and one successful method is to centrifuge oil from the hollow main shaft, collecting it in a labyrinth arrangement and guiding it into the hollow planet pin and out of a suitably placed hole into the roller bearing.

CASING

The design of the casings for epicyclic gear boxes is relatively simple since tooth loads are balanced and bearing loads on the main elements are nominal. For this reason it is possible and usual to use aluminium alloy casings and, on transmissions intended for large volume production, these should invariably be designed as pressure die castings.

NOISE

The principal source of noise is deformation as teeth enter and leave engagement. A study of gear noise reduction should concentrate on gear design to reduce noise at source, but it should be remembered that resonance of some other part of the vehicle may magnify, to unacceptable levels, noises which are small in themselves.

Gear details, such as module, pressure angle, etc. affect noise as do bearing supports, eccentricities and out of parallelism. Lower modules, 1 to 3 are quite common. Support bearings for each member of the gear train have an influence on noise. With perfectly made parts, the aim should be absolute concentricity of the three elements. Experience has shown that some freedom of two of the elements to allow them to centralize to the well supported third element is a more practical solution.

Epicyclic gears of modern design have a good record of silence in operation - when noise is a problem, it is often due to characteristics of the vehicles and such problems are almost always difficult to diagnose and cure.

APPENDIX B

SPEEDS IN AN EPICYCLIC TRAIN

The various possible combinations in an epicyclic gear train are described below with the help of an example.

1. In most cases, the sun gear A is driven from the engine and the carrier of the planet gear D is the output member conveying the drive to the next transmission member. The outer cylindrical surface of the annulus gear is provided with a band brake, such that when this brake is applied, the annulus gear is locked. Assuming that the annulus (B) is locked and power applied to the sun gear Fig. (i), then the carrier (D) will revolve, carrying with it the planet gear (C). If the gear (A) rotates in a clockwise direction, (C) will rotate anti-clockwise and the carrier (D) will move in a clockwise direction also. Therefore if the carrier is a power output member it will rotate in the same direction as the sun gear but at a slower speed than the sun gear.

The speed of rotation of the planet carrier can be shown to be $\frac{A}{A+B}$ times the speed of the sun gear (A) where A and B denote the number of teeth of gears (A) and (B).

2. If the carrier (D) is locked, as by a clutch unit, and the sun gear is power-driven member, then as shown in Fig.(ii), if (A) revolves clockwise, it will rotate the planet gear (C) anticlockwise and this gear will in turn rotate the annulus

gear anticlockwise and at $\frac{A}{B}$ times the speed of (A).

3. If the sun gear (A) is locked and the carrier (D) is driven in a clockwise direction Fig.(iii) then the annulus gear (B) will be driven by the planet gear (C) in a clockwise direction and at a higher speed than (A), the ratio of the annulus gear to carrier speeds being $\frac{A+B}{B}$.

4. If the annulus gear (B) is locked and the carrier (D) becomes the clockwise direction drive Fig.(iv) it will rotate the planet gear C in an anticlockwise direction and sun gear (A) in a clockwise direction and at a higher speed than its own. Thus the sun gear will rotate at $\frac{A+B}{B}$ times the carrier speed.

5. If the carrier D is locked, the annulus be power driven and the drive is obtained from the sun gear which will rotate at $\frac{B}{A}$ times the speed of (A).

6. Again if the sun gear A is locked and annulus gear B is the driving gear, the carrier D will rotate at a slower speed given by $\frac{B}{A+B}$ times that of the speed of B.

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```
C     PROGRAMME FOR COMPUTER AIDED DESIGN
 20   FORMAT(19X,*SUN TEETH*,7X,*ANNULUS TEETH*,3X,*PLANET TEETH*,3X,*NO
     1 OF PLANETS*,5X,*GEAR RATIO*,6X,*MODULE*,6X,*ANNULUS DIAMETER*)
      PRINT20
      F1=6300.0
      RPM=1100.0
      T1=62.1
      SK=3.5
      FST=F1*100.0/7.0
      AC=0.0033
      AX=-0.0009/1000.0
      PI=4.*ATAN(1.)
      XR1=4.28
C      THE NUMBER OF TEETH ON SUN GEAR ARE MADE TO VARY FROM16 UPWARDS
      DO 45 I=1,20
      AI=I-1
      S1=16.0+AI
C      CALCULATION FOR THE NUMBER OF TEETH FOR ANNULUS ANDPLANET GEARS
      A1=(XR1-1.0)*S1
      P1=(XR1-2.0)*S1/2.0
C      THE NUMBER OF TEETH ARE ROUNDED TO THE NEARESTINTEGER VALUES
      NS1=S1
      NA1=A1+0.5
      NP1=P1+0.5
      SN1=NS1
      AN1=NA1
      PN1=NP1
C      CALCULATION FOR LEWIS FORM FACTOR
      Y1=0.154-0.912/SN1
C      NEIGHBOURHOOD CONDITION APPLIED TO CALCULATE MAXIMUM NUMBER
      R1=(PN1+2.0)/(SN1+PN1)                                OF PLANETS
      THETA1=ARSIN(R1)
      PMAX1=PI/THETA1
      NPMAX1=PMAX1+0.5
      PMAXA1=NPMAX1
      DO 40 N=1,6
      BN=N-1
      PNX(N)=PMAXA1-BN
      IF(PNX(N).LT.2.0)GO TO 40
C      CHECK FOR ASSEMBLY OF GEARS IN THE FIRST GEAR TRAIN
      FCTOR1=(AN1+SN1)/PNX(N)
      IFAC=FCTOR1
      FAC=IFAC
      IF(FCTOR1-FAC-0.0000001)25,40,40
 25   NP=PNX(N)
      PT1=NP
C      SELECTION OF MODULE FROM THE PREFERRED VALUES TO SATISFY
C      MAXIMUM ALLOWABLE STRESS
      DO 90 M=1,10
      AM=M
      AM1=AM/2.0+1.0
      DIAS1=AM1*SN1
      DIAA1=AM1*AN1
      IF(DIAA1.GT.260.0)GO TO 90
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```
      VELS1=PI*DIAS1*RPM/25.4/12.0
      W1=2.0*T1/DIAS1*1000.0*2.2/PT1
      B1=PI*AM1*SK/25.4
      ERR1=AX*VELS1+AC
      C1=1660.0*ERR1/0.001
      WB1=FST*B1*Y1*PI*AM1/25.4
      WD1=0.05*VELS1*(B1*C1+W1)/(0.05*VELS1+SQRT(B1*C1+W1))+W1
70    PRINT75,NS1,NA1,NP1,NP,XR1,AM1,DIAA1
75    FORMAT(10X,4(11X,I5),3F16.2)
      PRINT77
77    FORMAT(2(1H))

C     CALCULATIONS FOR THE SIZE OF SECOND GEAR TRAIN

      DO 250 L=1,10
      AL=L
      XR2=2.39+AL/100.
      SN2=SN1
      NS2=SN2
C     CALCULATION FOR THE NUMBER OF TEETH FOR ANNULUS ANDPLANET GEARS
      A2=(SN1*XR1*(1.0-XR2))/(XR2-XR1)
      P2=(A2-SN2)/2.0
C     THE NUMBER OF TEETH ARE ROUNDED TO THE NEARESTINTEGER VALUES
      NA2=A2+0.5
      NP2=P2+0.5
      AN2=NA2
230   PN2=NP2
C     CALCULATION FOR LEWIS FORM FACTOR
      Y2=0.154-0.912/SN2
C     NEIGHBOURHOOD CONDITION APPLIED TO CALCULATE MAXIMUM NUMBER
C                                                      OF PLANETS
      R2=(PN2+2.0)/(SN2+PN2)
      THETA2=ARSIN(R2)
      PMAX2=PI/THETA2
      NPMAX2=PMAX2
      PMAX2=NPMAX2
      DO 240 NB=1,6
      BN=NB-1
      P2X(N)=PMAX2-BN
      IF(P2X(N).LT.2.0)GO TO 240
C     CHECK FOR ASSEMBLY OF GEARS IN THE FIRST GEAR TRAIN
      FCTOR2=(AN2+SN2)/P2X(N)
      IFAC=FCTOR2
      FAC=IFAC
      IF(FCTOR2-FAC-0.0000001)225,240,240
225   PT=P2X(N)
      NPT=PT
      PT2=NPT
C     SELECTION OF MODULE FROM THE PREFERRED VALUES TO SATISFY
C     MAXIMUM ALLOWABLE STRESS
      DO 290 MA=1,10
      BM=MA
      AM2=BM/2.0+1.0
      DIAS2=AM2*SN2
      DIAA2=AM2*AN2
C     THE DIAMETER OF THE SECOND ANNULUS IS TAKEN NEARLY EQUAL TO
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C      THE DIAMETER OF THE FIRST ANNULUS FOR COMPACTNESS
       IF(DIAA2.GT.0.99*DIAA1.AND.DIAA2.LT.1.01*DIAA1)GO TO 235
       GO TO 290
C      CHECK FOR STRENGTH OF GEAR TOOTH DUE TO DYNAMIC LOAD USING
C      BUCKINGHAM EQUATION
 235   VELS2=PI*DIAS2*RPM/25.4/12.0
       W2=2.0*T1/DIAS2*RPM*2.2/PT2
       B2=PI*AM2*SK/25.4
       ERR2=AX*VELS2+AC
       WB2=FST*B2*Y2*PI*AM2/25.4
       WD2=0.05*VELS2*(B2*C1+W2)/(0.05*VELS2+SQRT(B2*C1+W2))+W2
       IF(WB2-1.35*WD2)290,270,270
 270   PRINT275,NS2,NA2,NP2,NPT,XR2,AM2,DIAA2
 275   FORMAT(10X,4(11X,I5),3F16.2)
 290   CONTINUE
 240   CONTINUE
 250   CONTINUE
       PRINT277
 277   FORMAT(2(1H0))
       DO 350 MC=1,6
       XR3=1.56+AM/100.
C      *CALCULATION OF THE NUMBER OF TEETH OF ANNULUS AND PLANET GEARS
       G11=AN2*(AN1+SN1)-AN2*SN1*XR3
       G12=(AN1+SN1)*(AN2+SN2)-XR3*AN1*SN1-XR3*SN1*AN2-XR3*SN1*SN1
       G3=G11/G12
       DO 345 NC=1,10
C      THE NUMBER OF TEETH ON SUN GEAR ARE MADE TO VARY FROM16UPWARDS
       AN=NC-1
       AM=MC
       S3=16.0+AN
C      *CALCULATION OF TH5 NUMBER OF TEET8 OF ANNULUS AND PLANET GEARS
       A3=S3/(G3-1.0)
       P3=(A3-S3)/2.0
C      THE NUMBER OF TEETH ARE ROUNDED TO THE NEARESTINTEGER VALUES
       NS3=S3
       NA3=A3+0.5
       NP3=P3+0.5
       SN3=NS3
       PN3=NP3
       IF(PN3.LE.15.0)GO TO 345
       SN3=NS3
       AN3=NA3
C      CALCULATION FOR LEWIS FORM FACTOR
       Y3=0.154-0.912/SN3
C      NEIGHBOURHOOD CONDITION APPLIED TO CALCULATE MAXIMUM NUMBER
       R3=(PN3+2.0)/(SN3+PN3)                          OF PLANETS
       THETA3=ARSIN(R3)
       PMAX3=PI/THETA3
       NPMAX3=PMAX3
       PMAX3=NPMAX3
       DO 340 KA=1,6
       BK=KA
       PX(KA)=PMAX3-BK
       IF(PX(KA).LT.2.0)GO TO 340
C      CHECK FOR ASSEMBLY OF GEARS IN THE FIRST GEAR TRAIN
```

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      FCTOR3=(AN3+SN3)/PX(KA)
      IFAC=FCTOR3
      FAC=IFAC
      IF(FCTOR3-FAC-0.0000001)325,340,340
 325  NT3=PX(KA)
      PT3=NT3
C      SELECTION OF MODULE FROM THE PREFERRED VALUES TO SATISFY MAXIMUM
C      ALLOWABLE STRESS
      DO 390 MD=1,10
      DM=MD
      AM3=DM/2.0+1.0
      DIAS3=AM3*SN3
      DIAA3=AM3*AN3
      IF(DIAA3.GT.0.90*DIAA1.AND.DIAA3.LT.0.93*DIAA1)GO TO 335
      GO TO 390
C      THE DIAMETER OF THE FIRST ANNULUS FOR COMPACTNESS
C      BUCKINGHAM EQUATION
 335  VELS3=PI*DIAS3*RPM/25.4/12.0
      W3=2.0*T1/DIAS3*1000.0!2.2/PT3
      B3=PI*AM3*SK/25.4
      ERR3=AX*VELS3+AC
      WD3=0.05*VELS3*(B3*C1+W3)/(0.05*VELS3+SQRT(B3*C1+W3))+W3
      WB3=FST*B3*Y3*PI*AM3/25.4
      IF(WB3-1.35*WD3)390,370,370
 370  PRINT375,NS3,NA3,NP3,NT3,XR3,AM3,DIAA3
 375  FORMAT(10X,4(11X,I5),3F16.2)
 390  CONTINUE
 340  CONTINUE
 345  CONTINUE
 350  CONTINUE
      PRINT377
 377  FORMAT(2(1H0))
C      CALCULATIONS FOR THE SIZE OF REVERSE GEAR TRAIN

      DO+145 ND=1,20
      DN=ND-1
      SR=20.0+DN
C     *CALCULATION OF THE NUMBER OF TEETB OF ANNULUS AND PLANET GEARS
      AR=ABS(SN1*SR*(XR4-1.0)/AN1)
      PR=(AR-SR)/2.0
C      THE NUMBER OF TEETH ARE ROUNDED TO THE NEARESTINTEGER VALUES
      NSR=SR
      NAR=AR+0.5
      NPR=PR+0.5
      SNR=NSR
      ANR=NAR
      PNR=NPR
      IF(PNR.LE.15.0)GO TO 145
C      CALCULATION FOR LEWIS FORM FACTOR
      YR=0.154-0.912/SNR
C      NEIGHBOURHOOD CONDITION APPLIED TO CALCULATE MAXIMUM NUMBER OF
                                                            PLANETS
      RR=(PNR+2.0)/(SNR+PNR)
```

```
      THETAR=ARSIN(RR)
      PMAXR=PI/THETAR
      NPMAXR=PMAXR
      PMAXR=NPMAXR
      DO 140 KC=1,6
      CK=KC
       PRX(K)=PMAXR-CK
      IF(PRX(K).LT.2.0)GO TO 140
       FCTORR=(ANR+SNR)/PRX(2)
      IFAC=FCTORR
      FAC=IFAC
      IF(FCTORR-FAC-0.0000001)125,140,140
  125 NTR=PRX(K)
      PTR=NTR
C      SELECTION OF MODULE FROM THE PREFERRED VALUES TO SATISFY MAXIM
C      ALLOWABLE STRESS
      DO 190 MR=1,10
      RM=MR
      AMR=RM/2.0+1C0
      DIASR=AMR*SNR
      DIAAR=AMR*ANR
      IF(DIAAR.GT.0.99*DIAA1.AND.DIAAR.LT.1.01*DIAA1)GO TO 135
      GO TO190
C      CHECK FOR STRENGTH OF GEAR TOOTH DUE TO DYNAMIC LOAD USING
C      BUCKINGHAM EQUATION
  135 VELSR=PI*DIASR*RPM/25.4/12.0
      WR=2.0*T1/DIASR*1000.0*2.2/PTR
      BR=PI*AMR*SK/25.4
      ERRR=AX*VELSR+AC
      CR=1660.0*ERRR/0.001
      WDR=0.05*VELSR*(BR*CR+WR)/(0.05*VELSR+SCRT(BR*CR+WR))+WR
      WBR=FST*BR*YR*PI*AMR/25.4
      IF(WBR-1.35*WDR)190,170,170
  170 PRINT175,NSRTNAR,NPR,NTR,XR4,AMR,DIAAR
  175 FORMAT(10X,4(11X,I5),3F16.2)
  190 CONTINUE
  140 CONTINUE
  145 CONTINUE
  150 CONTINUE
         PRINT  177
  177 FORMAT(2(1H0))
   90 CONTINUE
   40 CONTINUE
   45 CONTINUE
      STOP

$ENTRY
```

```
C     PROGRAMME FOR PENALTY FUNCTION APPROACH
      DIMENSION B(12)
      COMMON/APT/PI,T,TE,FS,XR1,XR2,XR3,XR4,FEND,AN
      PI=4.0*ATAN(1.0)
      FEND=63.0
      AN=1100.0
      T=62100.0
      TE=1.5*T
      READ 100,(B(I),I=1,12)
  100 FORMAT(12F6.2)
      CALL FMIN(B)
      STOP
      END
$IBFTC  FMIN
      SUBROUTINE  FMIN(B)
      DIMENSION B(12),G(25),GW(25),GP(25),DELX(25),GW1(25),GP1(25),S(
     1,Y(25),HY(25),H(25,25),YK(25,25),AE(25,25),Y+(25,25),HYH(25,25)
     2(12,12),GF(12,12),GF1(12),DEL(12),DEL1(12)
      COMMON/APT/PI,T,TE,FS,XR1,XR2,XR3,XR4,FEND,AN
      COMMON/BPT/AN1,AN2,AN3,AN4,ZP1,ZP2,ZP3,ZP4
 7    FORMAT(1X,13E9.2)
 80   FORMAT(* PENALTY*,E10.2)
 108  FORMAT(20X,*THE PRESENT VALUE OF R=*,E12.6)
 102  FORMAT(1X,12E10.4,2X)
 101  FORMAT(20X,*THE INIATIAL VALUE OF R=*,E12.5)
 100  FORMAT(1X,10(E10.4,2X))
 107  FORMAT(20X,*THE NUMBER CF ITERATIONS=*,I4)
 110  FORMAT(/10X,*THESE ARE THE VALUES OF CONSTRAINTS*)
 112  FORMAT(/10X,*THES ARE THE VALUES OF VRIABLES OBJECT FUNCTION AN
     1ENALITY FUNCTION*)
 104  FORMAT(/10X,*THESE ARE THE VALUES OF GRADIENTS*)
 109  FORMAT(/10X,*THESE ARE THE VALUES OF DIRECTIONS*)
 38   CALL OFUNC(B,W,G,P)
 10   FORMAT(//10X,*AN1=*,F8.3)
 11   FORMAT(//10X,*AN2=*,F8.3)
 12   FORMAT(//10X,*AN3=*,F8.3)
 13   FORMAT(//10X,*AN4=*,F8.3)
 14   FORMAT(//10X,*ZP1=*,F8.3)
 15   FORMAT(//10X,*ZP3=*,F8.3)
 16   FORMAT(//10X,*ZP4=*,F8.3)
      PRINT110
      PRINT7,(G(I),I=1,25)
      PRINT 80,P
      PRINT10,AN1
      PRINT11,AN2
      PRINT12,AN3
      PRINT13,AN4
      PRINT14,ZP1
      PRINT15,ZP3
      PRINT16,ZP4
      IF(P.GT.10.**24)STOP
      CALL GRAD(B,GW,GP,G)
      SUM1=0.0
      SUM2=0.0
      DO 31 I=1,12
```

```
      SUM1=SUM1+GP(I)*GW(I)
   31 SUM2=SUM2+GP(I)*GP(I)
      R=-SUM1/SUM2
      IF(R.LT.0.0)R=0.5
      PRINT106,R
      IF(R) 32,33,34
   32 R=0.
      F=W+R*P
      F1=F
      PRINT102,(GW(I),I=1,12)
      DO 35 I=1,12
   35 DELX(I)=-GW(I)
      PRINT102,(DELX(I),I=1,12)
      SUM=0.0
      DO 36 I=1,12
   36 SUM=SUM+DELX(I)**2
      DO 37 I=1,12
   37 DELX(I)=DELX(I)/SQRT(SUM)
      CALL STEP(B,F,G,F1,DELX,ALP,R,W)
      PRINT103,ALP
      GO TO 38
   33 PRINT100,(B(I),I=1,12),W,F
      GO TO 50
   34 PRINT101,R
      KE=0.0
   23 DO 39 I=1,12
      F=W+R*P
   39 DEL(I)=GW(I)+R*GP(I)
      DO 41 I=1,12
      DO 41 J=1,12
   41 H(I,J)=0.0
      DO 42 I=1,12
   42 H(I,I)=1.0
      N=0.
   22 YHY=0.0
      STY=0.0
      DO 43 I=1,12
      DO 43 J=1,12
      HY(I)=0.0
      YH(I,J)=0.0
   43 HYH(I,J)=0.0
      DO 44 I=1,12
   44 DELX(I)=0.0
      DO 45 I=1,12
      DO 45 J=1,12
   45 DELX(I)=DELX(I)-H(I,J)*DEL(J)
      PRINT112
      PRINT100,(B(I),I=1,12),W,F
      PRINT104
      PRINT102,(DELX(I),I=1,12)
      PRINT109
      PRINT102,(DEL(I),I=1,12)
      SUM=0.0
      DO 49 I=1,12
   49 SUM=SUM+DELX(I)**2
```

```
      DO 51 I=1,12
51    4E3X(I)=DELXUI)/SQ9T(SUM)
      F1=F
      F2=F
      CALL STEP(B,F,G,F1,DELX,ALP,R,W)
      PRINT103,ALP
103   FORMAT(10X,*THE STEP LENGTH=*,E12.6)
      DO 56 I=1,12
56    S(I)=ALP*DELX(I)
      CALL GRAD(B,GW1,GP1,G)
      DO 57 I=1,12
      DEL1(I)=GW1(I)+R*GP1(I)
57    Y(I)=DEL1(I)-DEL(I)
      DO 58 I=1,12
58    STY=STY+S(I)MY(I)
      IF(ALP-0.0001)67,67,61
61    IF(SQRT(SUM).LE.0.01)GO TO 67
      DO 59 I=1,12
      DO 59 J=1,12
59    HY(I)=HY(I)+H(I,J)*Y(J)
      DO 60 I=1,12
60    YHY=YHY+Y(I)*HY(I)
      DO 62 I=1,12
      DO 62 J=1,12
      YK(I,J)=Y(I)*Y(J)
62    AE(I,J)=S(I)MS(J)/STY
      DO 63 I=1,12
      DO 63 J=1,12
      DO 63 K=1,12
63    YH(I,J)=YH(I,J)+YK(I,K)*H(K,J)
      DO 64 I=1,12
      DO 64 J=1,12
      DO 64 K=1,12
64    BE(I,J)=HYH(I,J)/YHY
      DO 65 I=1,12
      DO 65 J=1,12
65    H(I,J)=H(I,J)+AE(I,J)-BE(I,J)
      DO 66 I=1,12
      GW(I)=GW1(I)
      GP(I)=GP1(I)
66    DEL(I)=DEL1(I)
      N=N+1
      IF(N-100)68,68,67
68    IF(F2-F1)22,67,22
67    R=0.25*R
      PRINT106,R
106   FORMAT(20X,*THE PRESENT VALUE OF R=*,E12.6)
      KE=KE+1
      PRINT107,KE
      IF(KE-30)23,23,50
50    RETURN
      END
$IBFTC   OFUNC
      SUBROUTINE  OFUNC(B,W,G,P)
      DIMENSION B(12),G(25)
```

```
      COMMON/APT/PI,T,TE,FS,XR1,XR2,XR3,XR4,FEND,AN
      COMMON/BPT/AN1,AN2,AN3,AN4,ZP1,ZP2,ZP3,ZP4
      PN1=5.0
      PN3=3.0
      PN4=4.0
      P1=0.912/0.154
      P2=2.0*T/(0.154*PI**2*FEND)
      FB=3.5
      PS=PI**2/4.0
      A1=B(1)**3
      A2=B(2)**3
      A3=B(3)**3
      A4=B(4)**3
      Z1=(B(5)-1.0)**2
      Z2=((1.0-B(6))/(B(6)-B(5)))**2*B(5)**2
      Z3=((B(6)-B(5))*(B(5)*B(7)*(1.0+B(5))-(B(7)+B(5)))/(B(7)*(B(5)*B
     1)-2.0*B(5)*B(6)+B(6))-B(5)))**2
      Z4=(B(8)+1.0)**2/Z1
      W=PS*FB*A1*Z1*B(9)**2+PS*FB*A2*Z2*B(9)**2+PS*A3*FB*Z3*B(11)**2+P
     1FB*A4*Z4*B(12)**2
      W=W/(10.0**6)
      AN1=(B(5)-1.0)*B(9)
      AN1=ABS(AN1)
      C22=AN1/(AN1+B(9)-B(6)*B(9))
      AN2=B(10)*C22*B(6)-B(10)
      AN2=ABS(AN2)
      C33=(B(7)*AN1*B(9)-AN1*(AN1+B(9)))/(B(7)*B(9)*(2.0*AN1+B(9))-(AN
     1B(9))**2)
      C33=ABS(C33)
      AN3=B(11)/(C33-1.0)
      AN4=ABS(B(9)*B(12)*(B(8)+1.0)/AN1)
      ZP1=(AN1-B(9))/2.0
      ZP3=(AN3-B(11))/2.0
      ZP4=(AN4-B(12))/2.0
      DEN1=B(9)+ZP1
      DEN3=B(11)+ZP3
      DEN4=B(12)+ZP4
      S1=(ZP1+2.0)/DEN1
      S3=(ZP3+2.0)/DEN3
      S4=(ZP4+2.0)/DEN4
      S3=ABS(S3)
      S4=ABS(S4)
      S1=ABS(S1)
      S1=AMAX1(S1,S3,S4)
      C1=ARSIN(S1)
      C3=ARSIN(S3)
      C4=ARSIN(S4)
      CI1=PI/C1
      CI3=PI/C3
      CI4=PI/C4
      D1=ABS(AN1*B(1))
      D2=ABS(AN2*B(2))
      D3=ABS(AN3*B(3))
      D4=ABS(AN4*B(4))
      DS4=B(12)*B(4)
```

```
DS3=B(11)*B(3)
DS1=B(9)*B(1)
C=0.0033
X=-0.0009/1000.0
FST=FEND*10000.0/7.0
EL=PI*AN/25.4/12.0
CF=1666.0/0.001
DL=2.0*T*2.2
Y1=0.154-0.912/B(9)
VELS1=DS1*EL
W1=DL/DS1
W1=W1/PN1
B1=PI*FB*B(1)/25.4
ERR1=X*VELS1+C
CF1=CF*ERR1
WD1=0.05*VELS1*(B1*CF1+W1)/(0.05*VELS1+SQRT(B1*CF1+W1))+W1
WB1=FST*B1*Y1*PI*B(1)/25.4
Y3=0.154-0.912/B(11)
VELS3=DS3*EL
W3=DL/DS3
W3=W3/PN3
B3=PI*FB*B(3)/25.4
ERR3=X*VELS3+C
CF3=CF*ERR3
WD3=0.05*VELS3*(B3*CF3+W3)/(0.05*VELS3+SQRT(B3*CF3+W3))+W3
WB3=FST*B3*Y3*PI*B(3)/25.4
Y4=0.154-0.912/B(12)
VELS4=DS4*EL
W4=DL/DS4
W4=W4/PN4
B4=PI*FB*B(4)/25.4
ERR4=X*VELS4+C
CF4=CF*ERR4
WD4=0.05*VELS4*(B4*CF4+W4)/(0.05*VELS4+SQRT(B4*CF4+W4))+W4
WB4=FST*B4*Y4*PI*B(4)/25.4
G(1)=WB1-1.25*WD1
G(2)=CI1-3.0
G(3)=WB3-1.25*WD3
G(4)=CI3-3.0
G(5)=WB4-1.25*WD4
G(6)=CI4-3.0
G(7)=0.85*D1-D3
DIF1=ABS((D1-D4)/D1)
DIF2=ABS((D1-D2)/D1)
G(8)=0.15-DIF1
G(9)=0.15-DIF2
G(10)=B(1)-1.0
G(11)=B(2)-1.0
G(12)=B(3)-1.0
G(13)=B(4)-1.0
G(14)=B(5)-4.195
G(15)=4.36-B(5)
G(16)=B(6)-2.3814
G(17)=2.4786-B(6)
G(18)=B(7)-1.5582
```

```
      G(19)=1.6218-B(7)
      G(20)=B(8)-5.8306
      G(21)=6.1094-B(8)
      G(22)=B(9)-15.99
      G(23)=B(10)-15.99
      G(24)=B(11)-15.99
      G(25)=B(12)-16.0
      P=0.0
      DO 5I=1,25
 5    P=P+1.0/G(I)
      DO 4I=1,25
      IF(G(I).GT.0.0)GO TO 4
      P=10.0**25
       GO TO 6
 4    CONTINUE
 6    RETURN
      END
$IBFTC  GRAD
      SUBROUTINE  GRAD(B,GW,GP,G)
      DIMENSION B(12),G(25),GW(12),GP(12)
      COMMON/APT/PI,T,TE,FS,XR1,XR2,XR3,XR4,FEND,AN
      X=0.0005
      CALL OFUNC(B,W,G,P)
      X2=W
      X3=P
      B(1)=B(1)+X
      CALL OFUNC(B,W,G,P)
      X4=W
      X5=P
      GW(1)=(X4-X2)/X
      GP(1)=(X5-X3)/X
      B(2)=B(2)+X
      B(1)=B(1)-X
      CALL OFUNC(B,W,G,P)
      X6=W
      X7=P
      GW(2)=(X6-X2)/X
      GP(2)=(X7-X3)/X
      B(3)=B(3)+X
      B(2)=B(2)-X
      CALL OFUNC(B,W,G,P)
      X31=W
      X32=P
      GW(3)=(X31-X2)/X
      GP(3)=(X32-X3)/X
      B(4)=B(4)+X
      B(3)=B(3)-X
      CALL OFUNC(B,W,G,P)
      X41=W
      X42=P
      GW(4)=(X41-X2)/X
      GP(4)=(X42-X3)/X
      B(5)=B(5)+X
      B(4)=B(4)-X
      CALL OFUNC(B,W,G.P)
```

```
X51=W
X52=P
GW(5)=(X51-X2)/X
GP(5)=(X52-X3)/X
B(6)=B(6)+X
B(5)=B(5)-X
CALL OFUNC(B,W,G,P)
X61=W
X62=P
GW(6)=(X61-X2)/X
GP(6)=(X62-X3)/X
B(7)=B(7)+X
B(6)=B(6)-X
CALL OFUNC(B,W,G,P)
X71=W
X72=P
GW(7)=(X71-X2)/X
GP(7)=(X72-X3)/X
B(8)=B(8)+X
B(7)=B(7)-X
CALL OFUNC(B,W,G,P)
X81=W
X82=P
GW(8)=(X81-X2)/X
GP(8)=(X82-X3)/X
B(9)=B(9)+X
B(8)=B(8)-X
CALL.OFUNC(B,W,G,P)
X91=W
X92=P
GW(9)=(X91-X2)/X
GP(9)=(X92-X3)/X
B(9)=B(9)+X
B(8)=B(8)-X
CALL OFUNC(B,W,G,P)
X101=W
X102=P
GW(10)=(X101-X2)/X
GP(10)=(X101-X2)/X
B(11)=B(11)+X
B(10)=B(10)-X
CALL OFUNC(B,W,G,P)
X111=W
X112=P
GW(11)=(X111-X2)/X
GP(11)=(X112-X3)/X
B(12)=B(12)+X
B(11)=B(11)-X
CALL OFUNC(B,W,G,P)
X121=W
X122=P
GW(12)=(X121-X2)/X
GP(12)=(X122-X3)/X
RETURN
END
```

```
$IBFTC   STEP
      SUBROUTINE STEP(B,F,G,F1,DELX,AL
      DIMENSION B(12),DELX(12),G(25),ALP(12),Y(12),C(12),ALD(12)
      COMMON/APT/PI,T,TE,FS,XR1,XR2,XR3,XR4,FEND
      DO 10 I=1,12
      Y(I)=B(I)
 10   ALP(I)=-B(I)/DELX(I)
      KK=1
      AMIN=ALP(1)
 11   KK=KK+1
      IF(KK.GT.12)GO TO 12
      IF(AMIN.LT.ALP(KK))GO TO 11
      AMIN=ALP(KK)
      GO TO 11
 12   BETA=ABS(AMIN)
      ALPHA=0.5*BETA
      CL=ALPHA
      PRINT25,CL
 25   FORMAT(5X,E10.2)
      IF(CL.GT.1.0)CL=0.5
 22   PF=F
 24   DO 28 I=1,12
 28   C(I)=B(I)+CL*DELX(I)
      CALL OFUNC(C,W,G,P)
      F=W+R*P
      PRINT110,(C(I),I=1,12),W,F
 110  FORMAT(1X,7(E10.4,1X))
      IF(PF-F)31,31,32
 31   CL=0.5*CL
      IF(CL-0.0001*ALPHA)33,33,24
 32   DO 34 I=1,12
 34   B(I)=C(I)
      GO TO 22
 33   CALL OFUNC(B,W,G,P)
      F=W+R*P
      DO 40 I=1,12
 40   ALD(I)=(B(I)-Y(I))/DELX(I)
      N=1
      AMIN=ALD(1)
 41   N=N+1
      IF(N.GT.12)GO TO 42
      IF(AMIN.LT.ALD(N))GO TO 41
      AMIN=ALD(N)
      GO TO 41
 42   ALT=ABS(AMIN)
      IF(F1-F)35,35,36
 36   F1=F
 35   RETURN
      END
$ENTRY
  2.5   2.5   3.0   3.0   4.28  2.42  1.58  5.96 28.0  28.0  24.0  36.0
```